

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Determine which of the following statements regarding immunization are true:
  - A. Under full immunization, the present value of assets is greater than or equal to the present value of liabilities for small changes in interest rates.
  - B. Under Redington immunization, the duration of assets is equal to the duration of liabilities for all interest rates.
  - C. Under both Redington and full immunizations, the convexity of assets is greater than the convexity of liabilities for all interest rates.

(A) A only      (B) A and B      (C) B only      (D) B and C      (E) All are true

2. The current yield curve is flat at 5%. A liability of 22050, due in 5 years, is to be fully immunized using 3-year and 7-year zero coupon bonds. Determine the excess of assets over the liability at the time the liability is due, assuming that in 3 years the yield curve is flat at 3%.
  - (A) Less than 10
  - (B) Greater than or equal to 10 but less than 12
  - (C) Greater than or equal to 12 but less than 14
  - (D) Greater than or equal to 14 but less than 16
  - (E) Greater than or equal to 16

3. You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rates			Portfolio Rates
	$i_1^y$	$i_2^y$	$i_3^y$	
$y$	$i_1^y$	$i_2^y$	$i_3^y$	$i^{y+3}$
2006	0.10	0.06	0.05	$x$
2007	0.10	0.08	0.05	$x$
2008	0.10	0.05	0.04	$x$

1000 is invested on 01/01/2006 and another 1000 is invested on 01/01/2008 accumulates to the same amount on 12/31/2011 as if 1946 was invested on 01/01/2007. Determine  $x$ .

- (A) 3.00%      (B) 3.25%      (C) 3.50%      (D) 3.75%      (E) 4.00%

4. A three-year 1000 par value  $r\%$  annual coupon bond, redeemable at par, is bought to yield  $r\%$ . This yield is consistent with the current one-year, two year, and three year spot rates, which are 6%, 7%, and 8%, respectively. Determine  $r$ .

- (A) 6.0      (B) 7.3      (C) 7.9      (D) 8.2      (E) 8.5

5. You are given:

- (i) the annual yield rate on a zero-coupon bond with duration of 1 year is 3.0%
- (ii) the annual yield rate on a zero-coupon bond with duration of 2 years is 3.5%
- (iii) the annual yield rate on a zero-coupon bond with duration of 3 years is 4.0%
- (iv) the annual yield rate on a zero-coupon bond with duration of 4 years is 4.5%

The price, that is consistent with the above rates, for a 4-year 1000 face value bond, redeemable at  $C$ , with 5% annual coupons is 1088. Determine  $C$ .

- (A) 1100      (B) 1150      (C) 1200      (D) 1250      (E) 1300

6. You are given:

- (i) the current 1-year spot rate is 3.0%
- (ii) the current 2-year spot rate is 3.5%
- (iii) the current 3-year spot rate is 4.0%

Determine the minimum price that can be paid for a 5 year 10000 zero coupon bond that would be consistent with the above rates and the assumption that future spot rates remain the same as the above rates.

- (A) 8220      (B) 8300      (C) 8380      (D) 8420      (E) 8625

7. On January 1 a pension fund has a balance of  $\$X$ . A contribution of  $\$200,000$  is paid into the fund on a date prior to June 30. Immediately before the contribution, the balance in the fund is  $\$1,375,000$ . The balance on June 30 is  $\$1,732,500$  and on July 1 benefit payments of  $\$125,000$  are paid from the fund. The balance on December 31 is  $\$1,607,500$ . The time weighted and dollar weighted rates of return on the fund are both equal to 21%. Determine the month in which the contribution was made.

- (A) In February
- (B) In March
- (C) In April
- (D) In May
- (E) In June

8. A bond will pay monthly coupons of 10 at the end of each month for the next twenty years and will pay the face value of 1000 at the end of the twenty-year period.

Calculate the bond's Modified duration (in years) using a 12%, compounded monthly, interest rate.

- (A) 7.57      (B) 7.64      (C) 8.53      (D) 9.08      (E) 9.17

9. You are given:

- (i) the annual effective interest rate for the coming year (year one) is 4%
- (ii) the annual effective forward interest rate for year two is 5%
- (iii) the annual effective forward interest rate for year three is 6%

Determine ratio of the two year spot rate to the three year spot rate for spot rates that are consistent with these forward rates.

- (A) 0.900      (B) 0.995      (C) 1.005      (D) 1.050      (E) 1.111

10. You are given:

Bond A is a 1000 par value 10 year bond, redeemable at par, with annual coupons of 90. It has Macaulay duration of 7 years using an annual effective interest rate of 9%.

Bond B is a 13 year zero coupon bond.

Bond C is a  $n$ -year bond with a duration of 10.93 and price of 1000 using an annual effective interest rate of 9%

A portfolio consisting of these three bonds is created such that the Macaulay duration of the portfolio using an annual effective interest rate of 9% is 10.31? Determine the face amount of Bond B.

- (A) 1000      (B) 1785      (C) 2140      (D) 2895      (E) 3065

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Determine which of the following statements regarding immunization are true:

- A. Under full immunization, the present value of assets is greater than the present value of liabilities for small changes in interest rates.
- B. Under Redington immunization, the duration of assets is equal to the duration of liabilities for all interest rates.
- C. Under both Redington and full immunizations, the convexity of assets is greater than the convexity of liabilities for all interest rates.

(A) A only    (B) A and B    (C) B only    (D) B and C    (E) All are true

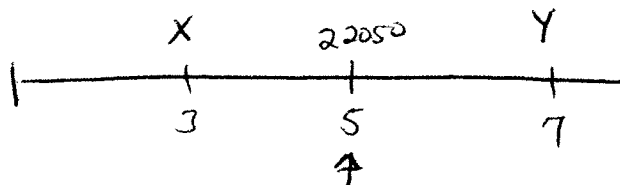
*A is true for all changes in interest rates, and so it is true for small changes as well.*

*B & C are true at the interest rate at which immunization is achieved, but not for all interest rates (Answer: A only)*

2. The current yield curve is flat at 5%. A liability of 22050, due in 5 years, is to be fully immunized using 3-year and 7-year zero coupon bonds. Determine the excess of assets over the liability at the time the liability is due, assuming that in 3 years the yield curve is flat at 3%.

- (A) Less than 10
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- (E) Greater than or equal to 16

*Let X = face amt. of 3-year bond  
Y = \_\_\_\_\_ 7 \_\_\_\_\_*



$e^{\delta} = 1.05$

$$\left. \begin{aligned} (1) \quad 22050 &= X e^{2\delta} + Y e^{-2\delta} \\ (2) \quad 0 &= 2X e^{2\delta} - 2Y e^{-2\delta} \end{aligned} \right\} \Rightarrow \begin{aligned} X e^{2\delta} &= Y e^{-2\delta} \text{ by (1)} \\ \therefore 22050 &= 2X e^{2\delta} = 2X (1.05)^2 \\ \Rightarrow X &= 10000 \end{aligned}$$

$$10000 (1.05)^2 = Y (1.05)^{-2} \Rightarrow Y = 12155$$

$$\therefore \text{Excess} = 10000 (1.03)^2 + 12155 (1.03)^{-2} - 22050 = 16.25$$

3. You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rates			Portfolio Rates
	$i_1^y$	$i_2^y$	$i_3^y$	$i^{y+3}$
2006	0.10	0.06	0.05	$x$
2007	0.10	0.08	0.05	$x$
2008	0.10	0.05	0.04	$x$

1000 is invested on 01/01/2006 and another 1000 is invested on 01/01/2008 accumulates to the same amount on 12/31/2011 as if 1946 was invested on 01/01/2007. Determine  $x$ .

- (A) 3.00%    (B) 3.25%    (C) 3.50%    (D) 3.75%    (E) 4.00%

$$1000(1.1)(1.06)(1.05)(1+x)^2 + 1000(1.1)(1.05)(1.04)(1+x) = 1946(1.1)(1.08)(1.05)(1+x)^2$$

$$\Rightarrow 1224.3(1+x)^2 - 2427.4404(1+x) + 1201.2 = 0$$

$$\Rightarrow 1+x = \frac{2427.4404 \pm 99.751}{2(1224.3)} \Rightarrow x = .0321$$

4. A three-year 1000 par value  $r\%$  annual coupon bond, redeemable at par, is bought to yield  $r\%$ . This yield is consistent with the current one-year, two year, and three year spot rates, which are 6%, 7%, and 8%, respectively. Determine  $r$ .

- (A) 6.0    (B) 7.3    (C) 7.9    (D) 8.2    (E) 8.5

The bond is bought at par.  $P=1000$

$$P=1000 = \frac{10r}{1.06} + \frac{10r}{(1.07)^2} + \frac{10r}{(1.08)^3} + \frac{1000}{(1.08)^3}$$

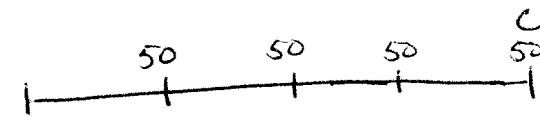
$$\Rightarrow r = 7.9$$

5. You are given:

- (i) the annual yield rate on a zero-coupon bond with duration of 1 year is 3.0%
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- (iv) the annual yield rate on a zero-coupon bond with duration of 4 years is 4.5%

The price, that is consistent with the above rates, for a 4-year 1000 face value bond, redeemable at  $C$ , with 5% annual coupons is 1088. Determine  $C$ .

- (A) 1100      (B) 1150      (C) 1200      (D) 1250      (E) 1300



$$P = 1088 = \frac{50}{1.03} + \frac{50}{(1.035)^2} + \frac{50}{(1.04)^3} + \frac{C+50}{(1.045)^4}$$

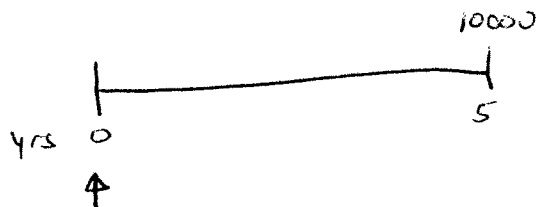
$$\Rightarrow C = 1080$$

6. You are given:

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Determine the minimum price that can be paid for a 5 year 10000 zero coupon bond that would be consistent with the above rates and the assumption that future spot rates remain the same as the above rates.

- (A) 8220      (B) 8300      (C) 8380      (D) 8420      (E) 8625



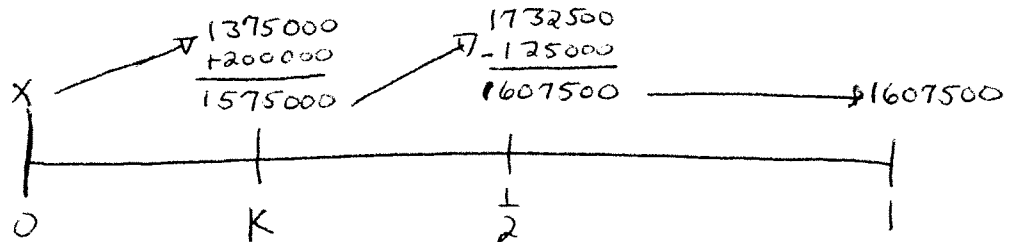
$P = \text{minimum when using 2-year and 3-year spot rates}$

$$\therefore P = \frac{10000}{(1.035)^2 (1.04)^3} = 8300$$



7. On January 1 a pension fund has a balance of \$X. A contribution of \$200,000 is paid into the fund on a date prior to June 30. Immediately before the contribution, the balance in the fund is \$1,375,000. The balance on June 30 is \$1,732,500 and on July 1 benefit payments of \$125,000 are paid from the fund. The balance on December 31 is \$1,607,500. The time weighted and dollar weighted rates of return on the fund are both equal to 21%. Determine the month in which the contribution was made.

- (A) In February
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$$i_{TW} = .21 \Rightarrow 1.21 = \frac{1375000}{X} \cdot \frac{1732500}{1575000} \cdot \frac{1607500}{1607500} \Rightarrow X = 1250000$$

$$i_{DW} = .21 \Rightarrow 1250000(1.21) + 200000(1 + (1-k)(.21)) - 125000(1 + \frac{1}{2}(.21)) = 1607500$$

$$\Rightarrow k = 0.2113 \Rightarrow 12k = 2.5 +$$

2.5 months after Jan 1 is in March.

8. A bond will pay monthly coupons of 10 at the end of each month for the next twenty years and will pay the face value of 1000 at the end of the twenty-year period.

Calculate the bond's Modified duration (in years) using a 12%, compounded monthly, interest rate.

- (A) 7.57
- (B) 7.64
- (C) 8.53
- (D) 9.08
- (E) 9.17

$$i = .01 \text{ meir} = r$$

$$\text{and } F = C$$

$$\therefore \text{Mod } D = a_{\overline{240}|.01} = 90.82 \text{ (months)}$$

$$\doteq 7.57 \text{ years}$$

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- (A) 0.900      (B) 0.995      (C) 1.005      (D) 1.050      (E) 1.111

$$(1+s_2)^2 = (1.04)(1.05)$$

$$(1+s_3)^3 = (1.04)(1.05)(1.06)$$

$$\Rightarrow \frac{s_2}{s_3} = \frac{\sqrt[2]{1.04(1.05)} - 1}{\sqrt[3]{1.04(1.05)(1.06)} - 1} \doteq 0.900$$

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Let  $X$  = face amount of Bond B

$$P_B = \text{Price of Bond B} = X v_{.09}^{13} \quad P_A = 1000 = P_C$$

$$\therefore 10.31 = \left( \frac{1000}{2000 + X v^{13}} \right) (7) + \left( \frac{X v^{13}}{2000 + X v^{13}} \right) (13) + \left( \frac{1000}{2000 + X v^{13}} \right) (10.93)$$

$$\Rightarrow X v_{.09}^{13} = 1000 \Rightarrow X \doteq 3065$$