Extra Problems for Test 3

1. Amy invests 1000 at an effective annual rate of 14% for 10 years. Interest is payable annually and is reinvested at an annual effective rate of i. At the end of 10 years the accumulated interest is 2341.08.

Bob invests 150 at the end of each year for 20 years at an annual effective rate of 15%. Interest is payable annually and is reinvested at an annual effective rate of i.

Find Bob's accumulated interest at the end of 20 years.

a. 9000 b. 9010 c. 9020 d. 9030 e. 9040

2. Jason deposits 3960 into a bank account at $t = 0$. The bank credits interest at the end of each year at a force of interest $\delta_t = \frac{1}{8+t}$.

Interest can be reinvested at an annual effective rate of 7%.

The total accumulated amount at time $t = 3$ is equal to X.

Calculate X.

a. 5394 b. 5465 c. 5551 d. 5600 e. 5685

3. The following table gives the pattern of investment year and portfolio interest rates over a three-year period, where $m = 2$ is the time after which the portfolio method is applicable.

Calendar Year of Original Investment, y	Investment Year Rates i_1^y	i_2^y	Portfolio Rates i^{y+2}	Calendar Year of Portfolio Rate, $y + 2$
Z	9.00%	10.00%	11.00%	$Z + 2$
$Z+1$	7.00	8.00		
$Z+2$	5.00			

Frank invests 1000 at the beginning of each of calendar years Z, $Z + 1$, and $Z + 2$. What is the total amount of interest credited to Frank's account for calendar year $Z + 2$.

a. 221.43 b. 233.67 c. 245.29 d. 256.19 e. 267.49

Extra Problems for Test 3

4. 1,000 is deposited into a fund on January 1, 1999. Another deposit is made into the fund on July 1, 1999. On January 1, 2000, the balance in the fund is 2,000.

The time-weighted yield rate is 10% and the dollar-weighted yield rate is 9%.

Calculate the annual effective interest rate earned on the fund during the first six months of 1999.

a. 5.4% b. 6.4% c. 9.4% d. 12.9% e. 13.3%

5. A 100,000 loan is to be repaid by 30 equal payments at the end of each year. The outstanding balance is amortized at 4%. In addition to the annual payments, the borrower must pay an origination fee at the time the loan is made. The fee is 2% of the loan but does not reduce the loan balance. When the second payment is due, the borrower pays the remaining loan balance. Determine the yield to the lender considering the origination fee and the early pay-off of the loan.

a. 4.9% b. 5.0% c. 5.1% d. 5.2% e. 5.3%

6. Sally lends 10,000 to Tim. Tim agrees to pay back the loan over 5 years with monthly payments payable at the end of each month.

Sally can reinvest the monthly payments from Tim in a savings account paying interest at 6%, compounded monthly. The yield rate earned on Sally's investment over the five-year period turned out to be 7.45%, compounded semiannually.

What nominal rate of interest, compounded monthly, did Sally charge Tim on the loan?

a. 8.53% b. 8.59% c. 8.68% d. 8.80% e. 9.16%

7. Paul lends 8000 to Peter. Peter agrees to pay it back in 10 annual installments at 7% with the first payment due in one year. After making 4 payments, Peter renegotiates to pay off the debt with 4 additional annual payments. The new payments are calculated so that Paul will get a 6.5% annual yield over the entire 8-year period. Determine how much money Peter saved by renegotiating.

a. Less than 550 b. At least 550, but less than 600 c. At least 600, but less than 650

d. At least 650, but less than 700 e. At least 700

Extra Problems for Test 3

8. A corporation borrows 10,000 for 25 years, at an effective annual interest rate of 5%. A sinking fund is used to accumulate the principal by means of 25 annual deposits earning an effective annual interest rate of 4%. Calculate the sum of the net amount of interest paid in the 13th installment and the increment in the sinking fund for the ninth year.

a. 664 b. 674 c. 684 d. 694 e. 704

9. A loan of 1000 is being repaid in ten years by semiannual installments of 50, plus interest on the unpaid balance at 4% per annum compounded semiannually. The installments and interest payments are reinvested at 5% per annum compounded semiannually. Calculate the annual effective yield rate of the loan.

a. .046 b. .048 c. .050 d. .052 e. .054

10. A loan of 10,000 is amortized by equal annual payments for 30 years at an annual effective interest rate of 5%. Determine the year in which the interest portion of the payment is most nearly equal to one-third of the payment.

a. 6 b. 7 c. 8 d. 23 e. 25

11. A loan of 1000 at a nominal rate of 12% convertible monthly is to be repaid by six monthly payments with the first payment due at the end of 1 month. The first three payments are X each, and the final three payments are $3X$ each. Determine the sum of the principal repaid in the third payment and the interest paid in the fifth payment.

a. 80 b. 82 c. 84 d. 86 e. 88

12. A 30-year loan of 1000 is repaid with payments at the end of each year.

Each of the first ten payment equals the amount of interest due. Each of the next ten payments equals 150% of the amount of interest due. Each of the last ten payments is X.

The lender charges interest at an annual effective rate of 10%.

Calculate X.

a. 32 b. 57 c. 70 d. 97 e. 117

Extra Problems for Test 3

13. A 1000 face value 20-year 8% bond with semiannual coupons is purchased for 1014. The redemption value is 1000. The coupons are reinvested at a nominal annual rate of 6%, compounded semiannually. Determine the purchaser's annual effective yield rate over the 20-year period.

a. 6.9% b. 7.0% c. 7.1% d. 7.2% e. 7.3%

14. A 9% bond with a 1000 par value and coupons payable semiannually is redeemable at maturity for 1100. At a purchase price of P, the bond yields a nominal annual interest rate of 8% compounded semiannually, and the present value of the redemption value is 190. Determine P.

a. 1050 b. 1085 c. 1120 d. 1165 e. 1215

15. A 1000 bond with annual coupons is redeemable at par at the end of 10 years. At a purchase price of 870, the yield rate is i. The coupon rate is $i - .02$.

Calculate i.

a. 6.7% b. 7.2% c. 7.7% d. 8.2% e. 8.7%

16. A bond with coupons equal to 40 sells for P. A second bond with the same maturity value and term has coupons equal to 30 and sells for Q. A third bond with the same maturity value and term has coupons equal to 80. All prices are based on the same yield rate, and all coupons are paid at the same frequency. Determine the price of the third bond.

a. $4P - 4Q$ b. $4P + 4Q$ c. $4Q - 3P$ d. $5P - 4Q$ e. $5Q - 4P$

17. An n-year 1000 par value bond with 8% annual coupons has an annual effective yield of i, $i > 0$. The book value of the bond at the end of year 3 is 1099.84 and the book value at the end of year 5 is 1082.27. Calculate the purchase price of the bond.

a. 1112 b. 1122 c. 1132 d. 1142 e. 1152

Extra Problems for Test 3

18. Becky buys an n-year 1000 par value bond with 6.5% annual coupons at a price of 825.44. The price assumes an annual effective yield rate of i. The total write-up in book value of the bond during the first 2 years after purchase is 23.76.

Calculate i. ($i > 0$)

a. 8.50% b. 8.75% c. 9.00% d. 9.25% e. 9.50%

19. A 30-year 10,000 bond that pays 3% annual coupons matures at par. It is purchased to yield 5% for the first 15 years and 4% thereafter. Calculate the amount for accumulation of discount for year 8.

a. 78 b. 83 c. 88 d. 93 e. 98

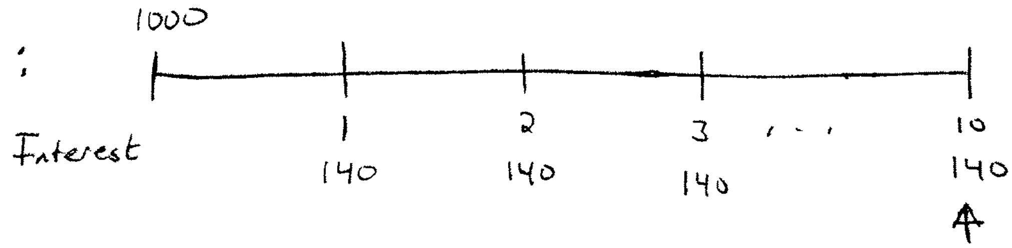
20. A 1000 par value 3-year bond, redeemable at par, with annual coupons of 50 for the first year, 70 for the second year, and 90 for the third year, is bought to yield a force of interest $\delta_t = \frac{2t-1}{2(t^2-t+1)}$ for $t > 0$. Calculate the price of this bond.

a. 500 b. 550 c. 600 d. 650 e. 700

21. A 1000 par value 4% bond with semiannual coupons matures at the end of 10 years. The bond is callable at 1050 at the ends of years 4 through 6, at 1025 at the end of years 7 through 9, and at 1000 at the end of year 10. Find the maximum price that an investor can pay in order to be certain of obtaining a yield rate of 5% convertible semiannually.

a. 922 b. 944 c. 959 d. 985 e. 1005

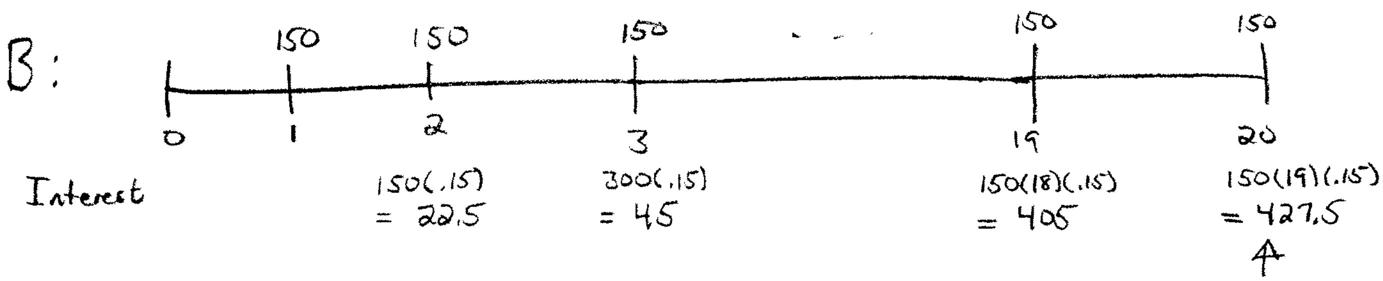
1) A:



$$\text{Accumulated Interest} = 2341.08 = 140\, s_{\overline{10|}\,i}$$

$$\Rightarrow i = 11\%$$

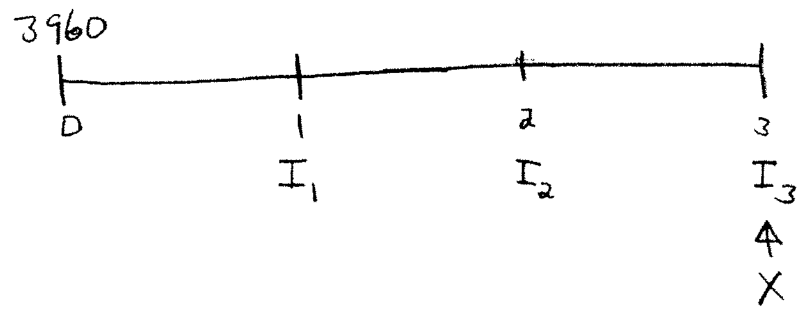
B:



$$\text{Accumulated Interest} = 22.5\,(Is)_{\overline{19|}\,11\%}$$

$$= 22.5 \cdot \frac{\ddot{s}_{\overline{19|}\,11\%} - 19}{.11} = 9041.49$$

2)



$$X = 3960 + I_1(1.07)^2 + I_2(1.07) + I_3$$

$$\left.\begin{aligned} I_1 &= 3960\, e^{\int_0^1 \frac{1}{8+t}\,dt} - 3960 \\ I_2 &= 3960\, e^{\int_1^2 \frac{1}{8+t}\,dt} - 3960 \\ I_3 &= 3960\, e^{\int_2^3 \frac{1}{8+t}\,dt} - 3960 \end{aligned}\right\}$$

$$e^{\int_K^{K+1} \frac{1}{8+t}\,dt} = e^{\ln(8+t)\big|_K^{K+1}} = \frac{9+K}{8+K}$$

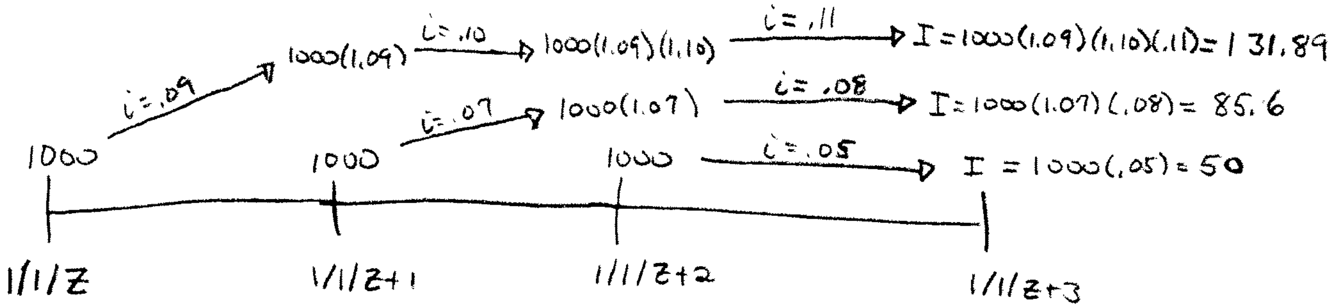
$$\therefore I_1 = 3960\left(\tfrac{9}{8}\right) - 3960 = 495$$

$$I_2 = 3960\left(\tfrac{10}{9}\right) - 3960 = 440$$

$$I_3 = 3960\left(\tfrac{11}{10}\right) - 3960 = 396$$

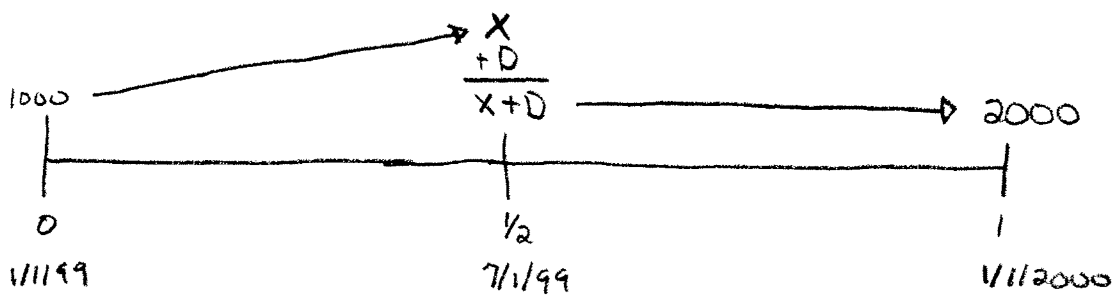
$$\therefore X = 3960 + 495(1.07)^2 + 440(1.07) + 396 = 5393.53$$

3)



$$I_{Total} = 131.89 + 85.6 + 50 = 267.49$$

4)



$$1+i_{TW} = \frac{X}{1000}\cdot\frac{2000}{X+D} \Longrightarrow 1.1 = \frac{2X}{X+D} \Longrightarrow D = \frac{9X}{11}$$

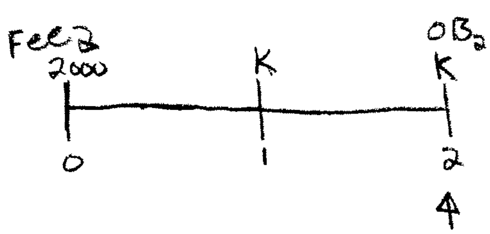
$$i_{DW} = .09 \Longrightarrow \quad 1000(1.09) + D\left(1+\tfrac{1}{2}(.09)\right) = 2000$$

$$\Longrightarrow D = 870.81$$

$$\therefore \quad X = 1064.33$$

$$\therefore \quad 1000(1+i)^{.5} = 1064.33 \Longrightarrow i \doteq 13.3\%$$

5)



$$100000 = K\, a_{\overline{30}|.04} \Rightarrow K = 5{,}783.01$$

$$OB_2 = K\, a_{\overline{28}|.04} \Longrightarrow OB_2 = 96{,}362.66$$

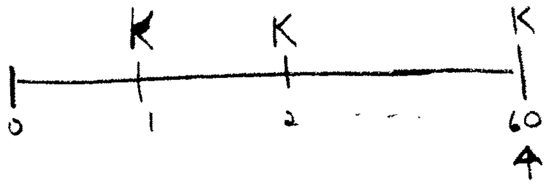
i = aeir = yield to lender

$$\therefore \quad 100000(1+i)^2 = 2000(1+i)^2 + K(1+i) + (K+OB_2)$$

$$\Longrightarrow 98000(1+i)^2 - 5783.01(1+i) - 102145.67 = 0$$

$$\Longrightarrow 1+i = \frac{5783.01 \pm \sqrt{(5783.01)^2 - 4(98000)(-102145.67)}}{2(98000)} \Rightarrow i \doteq .050864$$

6) Sally:



$$10000\left(1+\frac{.0745}{2}\right)^{10} = K\, s_{\overline{60}|.005}$$

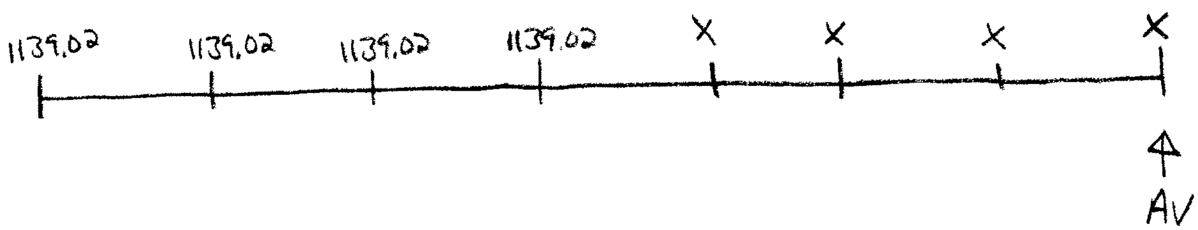
$$\Longrightarrow K = 206.62$$

Tim: $10000 = 206.62\, a_{\overline{60}|i} \Longrightarrow i = 0.733377 = \text{meir}$

$$i^{(12)} = 12i \doteq 8.80\%$$

7) Initially $8000 = K\, a_{\overline{10}|.07} \Longrightarrow K = 1139.02$

After negotiations:



$$AV = 8000(1.065)^8 = 1139.02\, s_{\overline{4}|6.5\%}(1.065)^4 + X\, s_{\overline{4}|6.5\%}$$

$$\Longrightarrow X = 1538.88$$

Initially Peter pays $10K = 10(1139.02) = 11390.20$

After negotiations, Peter pays $4K + 4X = 4(1139.02) + 4(1538.88)$

$= 10711.60$

Peter saves $11390.20 - 10711.60 = 678.60$
by renegotiating.

8) $K_{SF} \cdot s_{\overline{25}|.04} = 10000 \Rightarrow K_{SF} = 240.12$

$K_I = Li = 10000(.05) = 500$

Amount of Interest earned in SF during 13^{th} period

$= B_{12}^{SF} \cdot j = 240.12\, s_{\overline{12}|.04}(.04) = 144.32$

$\therefore$ net amount of interest paid in 13^{th} installment

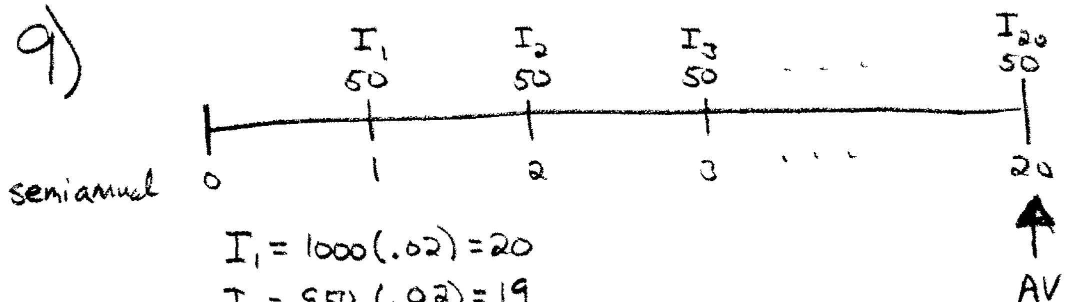
$= 500 - 144.32 = 355.68$

Increment in SF for 9^{th} year $= B_9^{SF} - B_8^{SF}$

$= 240.12\, s_{\overline{9}|.04} - 240.12\, s_{\overline{8}|.04} = 328.62$

$\therefore \sum = 355.68 + 328.62 = 684.30$

9)



$j = .025$ seir

$I_1 = 1000(.02) = 20$
$I_2 = 950(.02) = 19$
$I_3 = 900(.02) = 18$
$\vdots$
$I_{20} = 1$

$i =$ aeir $=$ yield rate

$AV = 1000(1+i)^{10} = 50\, s_{\overline{20}|.025} + (Ds)_{\overline{20}|.025}$

$= 1277.23 + \dfrac{20(1.025)^{20} - s_{\overline{20}|.025}}{.025}$

$= 1277.23 + 289.11 = 1566.34$

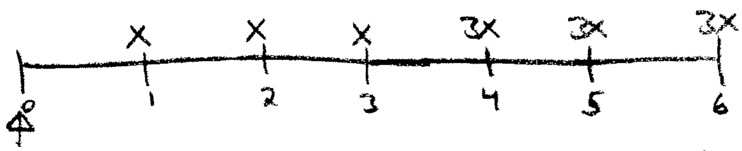
$\Rightarrow i = .046$

10) $I_t = \frac{1}{3}K \Rightarrow \cancel{K}\left(1 - v^{30-(t-1)}\right) = \frac{1}{3}\cancel{K}$

$\Rightarrow v^{31-t} = \frac{2}{3} \Rightarrow (1.05)^{31-t} = \frac{3}{2}$

$\Rightarrow t \doteq 22.6896$

11)



$1000 = 3X\, a_{\overline{6}|1\%} - 2X\, a_{\overline{3}|1\%} \Rightarrow X = 86.92$

Pr_3: $OB_2 = 1000(1.01)^2 - 86.92\, s_{\overline{2}|1\%} = 845.39$

$\Rightarrow I_3 = 845.39(.01) = 8.45$

$\Rightarrow Pr_3 = 86.92 - 8.45 = 78.47$

(OR $Pr_3 = OB_2 - OB_3$)

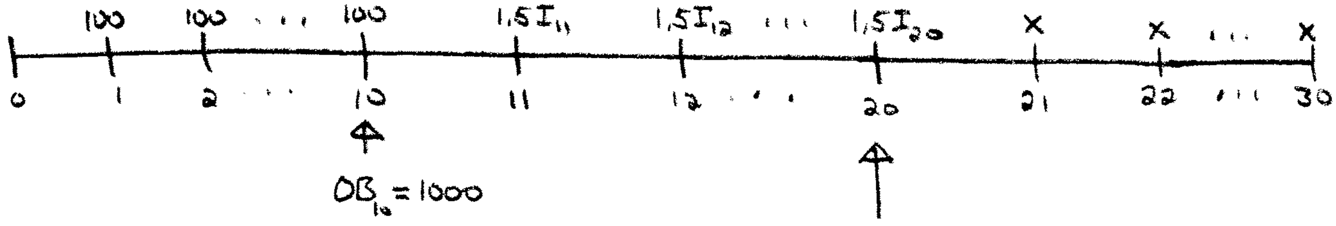
I_5: $OB_4 = 3X \cdot a_{\overline{2}|1\%} = 260.76\, a_{\overline{2}|1\%}$

$= 513.80$

$\Rightarrow I_3 = 513.80(.01) = 5.14$

$\therefore\ Pr_3 + I_5 = 78.47 + 5.14 = 83.61$

12)



$OB_{10} = 1000$

$OB_{11} = OB_{10}(1.1) - 1.5 \cdot OB_{10} \cdot (.1) = .95 \cdot (OB_{10})$

$OB_{12} = OB_{11}(1.1) - 1.5 \cdot OB_{11} \cdot (.1) = .95 \cdot OB_{11} = (.95)^2 \cdot (OB_{10})$

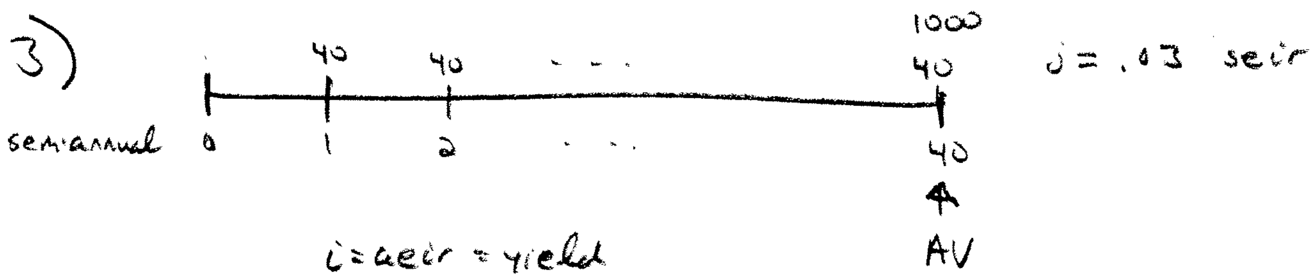
$OB_{13} = (.95)^3 \cdot (OB_{10})$

$\vdots$

$OB_{20} = (.95)^{10} \cdot (OB_{10}) = (.95)^{10}(1000) = X\, a_{\overline{10}|10\%}$

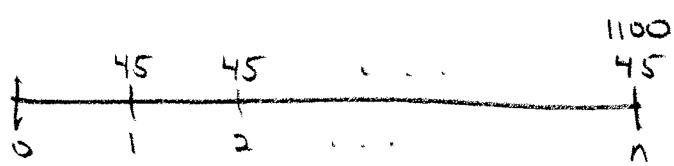
$\Rightarrow X = 97.44$

13)



$AV = 1014(1+i)^{20} = 1000 + 40\, s_{\overline{40}|.03} \Rightarrow i = 7.1\%$

14)



$P = 45\, a_{\overline{n}|.04} + 1100 v^n = \frac{45}{.04}(1-v^n) + 1100 v^n = 1125 - 25 v^n$

$190 = 1100 v^n \Rightarrow v^n = \frac{190}{1100}$

$\therefore P = 1125 - 25\left(\frac{190}{1100}\right) = 1120.68$

15) $870 = 1000(i - .02)\, a_{\overline{10}|i} + 1000\, v^{10}$

$= 1000\, i\, a_{\overline{10}|i} - 20\, a_{\overline{10}|i} + 1000\, v^{10}$

$= 1000\,(1 - v^{10}) - 20\, a_{\overline{10}|i} + 1000\, v^{10}$

$= 1000 - 20\, a_{\overline{10}|i}$

$\therefore\ 20\, a_{\overline{10}|i} = 1000 - 870 = 130 \Rightarrow i = 8.7\%$

16) $\left.\begin{array}{l} P = 40\, a_{\overline{n}|} + C v^n \\ Q = 30\, a_{\overline{n}|} + C v^n \end{array}\right\} \Rightarrow P - Q = 10\, a_{\overline{n}|}$

$C v^n = P - 40\, a_{\overline{n}|} = P - 4(P - Q)$
$= 4Q - 3P$

Price of 3rd bond $= R = 80\, a_{\overline{n}|} + C v^n$

$= 8 \cdot 10\, a_{\overline{n}|} + C v^n$

$= 8(P - Q) + (4Q - 3P) = 5P - 4Q$

17) $\underbrace{BV_3}_{PV}\,(1+i)^2 - \underbrace{80}_{PMT}\, s_{\underset{N}{\overline{2}}|i} = \underbrace{BV_5}_{FV} \Rightarrow i = 6.5\%$

$P = \underbrace{80}_{PMT}\, a_{\underset{N}{\overline{3}}|\underbrace{.065}_{I/Y}} + \underbrace{BV_3}_{FV} \cdot v^3 \Rightarrow P = 1122.38$

(OR $P = 80\, a_{\overline{5}|.065} + BV_5\, v^5$)

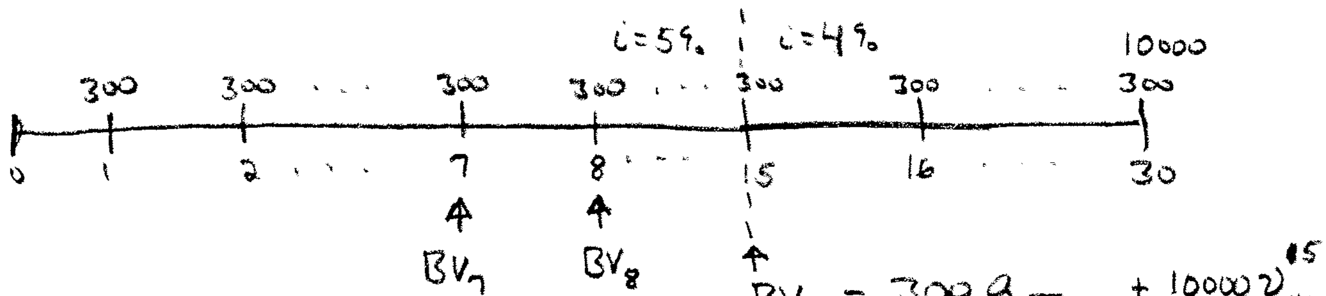
18) $P = BV_0 = 825.44$ $\qquad Fr = 65$

$$BV_2 = 825.44 + 23.76 = 849.2$$

$$\underbrace{825.44}_{PV} = \underbrace{65}_{PMT}\, a_{\overline{2}|i} + \underbrace{849.20\, v^2}_{FV} \Rightarrow i = 9.25\%$$

N

19)



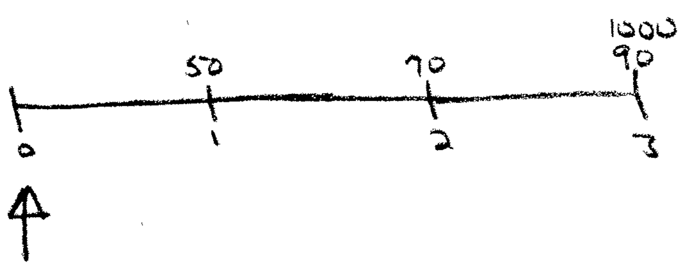
$$BV_{15} = 300\, a_{\overline{15}|.04} + 10000\, v_{.04}^{15} = 8888.16$$

$$BV_7 = 300\, a_{\overline{8}|5\%} + 8888.16\, v_{.05}^{8} = 7954.82$$

$$BV_8 = 300\, a_{\overline{7}|5\%} + 8888.16\, v_{.05}^{7} = 8052.56$$

$\Delta = 97.74 =$ accumulation of discount for year 8

20)



$$\delta_t = \frac{1}{2} \cdot \frac{2t-1}{t^2-t+1}$$

$$\Rightarrow a(t) = \left(\frac{t^2-t+1}{0^2-0+1}\right)^{1/2} = \sqrt{t^2-t+1}$$

$$P = PV = \frac{50}{a(1)} + \frac{70}{a(2)} + \frac{1090}{a(3)} = \frac{50}{1} + \frac{70}{\sqrt{3}} + \frac{1090}{\sqrt{7}} = 502.40$$

21)

	Call Time (in years)	Price to yield $i^{(2)} = .05$
$C=1050$	4	$P = 20\, a_{\overline{8}|.025} + 1050\, v_{.025}^{8} = 1005.19$
	5	$P = 995.30$
	6	$P = 985.89$
$C=1025$	7	$P = 20\, a_{\overline{14}|.025} + 1025\, v_{.025}^{14} = 959.24$
	8	$P = 951.57$
	9	$P = 944.26$
$C=1000$	10	$P = 20\, a_{\overline{20}|.025} + 1000\, v_{.025}^{20} = 922.05$

Note: If $P > 922.05$ and call time is $t=10$, then $i^{(2)} < .05$

$\therefore$ $P = 922.05$ is the maximum price that guarantees a yield of at least $i^{(2)} = .05$.