

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A bond with annual coupons of 40 has a book value of 859.06 just after the 5<sup>th</sup> coupon is paid and a book value of 891.62 just after the 14<sup>th</sup> coupon is paid. Determine the price, to the nearest dollar, at which this bond was bought.
  - (A) 840
  - (B) 843
  - (C) 846
  - (D) 849
  - (E) 852
  
2. A 10-year loan of 10,000 is repaid with annual payments using the sinking fund method. The sinking fund deposits earn an annual effective interest rate of 5% and the total annual payment is 1445.05. Determine the annual effective interest rate charged on the loan.
  - (A) 6.50%
  - (B) 6.75%
  - (C) 7.00%
  - (D) 7.25%
  - (E) 7.50%

3. A 10-year loan of 5000 is repaid with annual payments. The interest rate charged on the loan is 6% annual effective. Each annual payment equals 500 plus the interest due on the unpaid balance. When received, the payments are invested in an account that pays 5% annual effective. Determine the accumulated value at the end of the 10 years of the account in which the payments are invested.

(A) 6367

(B) 6996

(C) 7656

(D) 8039

(E) 8515

4. A 20-year loan is amortized with semiannual payments, with the first payment equal to 1000 and each subsequent payment being 2% greater than the preceding one. Using an annual effective interest rate of 4.04%, determine the amount of the 13<sup>th</sup> payment that repays principal.

(A) 570

(B) 610

(C) 640

(D) 670

(E) 700

5. A 250,000 mortgage is amortized for 30 years with monthly payments using a 4.8% compounded monthly interest rate. Determine the number of the payment for which the interest payment is less than twice the principal repaid for the first time.
- (A) 84
  - (B) 85
  - (C) 86
  - (D) 87
  - (E) 88
6. Two \$1000 par value bonds redeemable at par at the end of the same period are bought to yield 4% convertible quarterly. One bond costs \$1098 and has a coupon rate of 5% payable quarterly. The other bond has a coupon rate of 3.2% payable quarterly. Find the price of the second bond.
- (A) 892
  - (B) 902
  - (C) 912
  - (D) 922
  - (E) 932

7. A 10-year 1000 face value bond with 4% annual coupons and a redemption value of 1200 is bought to yield 4% annual effective. When received, each coupon is invested in an account that pays 5% annual effective. Immediately prior to receiving the 8<sup>th</sup> coupon, the investor sells the bond to a new buyer at a price to yield the new buyer 8%. Determine the annual yield rate the original investor received during the 8 years of ownership.
- (A) 3.04%
  - (B) 3.39%
  - (C) 3.57%
  - (D) 3.96%
  - (E) 4.27%
8. For a 5-year bond with semiannual coupons bought to yield 4% compounded semiannually, the amount of interest earned during the 4<sup>th</sup> installment is 5 more than the amount of the coupon. Determine which statement is true.
- (A) The bond is bought at a premium of 24.52
  - (B) The bond is bought at a discount of 24.52
  - (C) The bond is bought at a premium of 51.59
  - (D) The bond is bought at a discount of 51.59
  - (E) None of the above

9. A 30-year loan of 100,000 is amortized with level annual payments using an annual effective interest rate of 6%. Determine the total amount of interest paid on the loan during the middle 10 year period from time 10 years to time 20 years after loan inception.

(A) 39,200

(B) 40,100

(C) 41,000

(D) 41,900

(E) 42,800

10. A 1000 face value bond with 6% annual coupons is bought at par when bought to yield 5% annual effective. Determine the redemption value.

(A) 1000

(B) 1050

(C) 1100

(D) 1150

(E) 1200

MAP 4170  
Test 3

Name: KEY  
Date: July 16, 2012

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A bond with annual coupons of 40 has a book value of 859.06 just after the 5<sup>th</sup> coupon is paid and a book value of 891.62 just after the 14<sup>th</sup> coupon is paid. Determine the price, to the nearest dollar, at which this bond was bought.

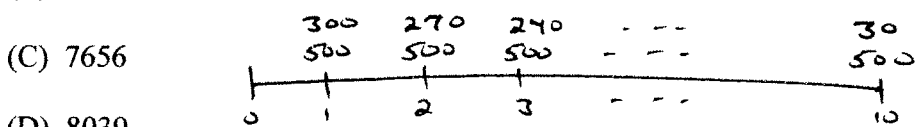
- (A) 840  $859.06 = 40 a_{\overline{9}|i} + 891.62 v^9$   
 (B) 843  $\Rightarrow i \doteq .05$   
 (C) 846  $P = 40 a_{\overline{14}|i} + 891.62 v_i^{14} \doteq 846.28$   
 (D) 849  
 (E) 852

2. A 10-year loan of 10,000 is repaid with annual payments using the sinking fund method. The sinking fund deposits earn an annual effective interest rate of 5% and the total annual payment is 1445.05. Determine the annual effective interest rate charged on the loan.

- (A) 6.50%  $R^{SF} \cdot s_{\overline{10}|.05} = 10000$   
 (B) 6.75%  $\Rightarrow R^{SF} \doteq 795.05$   
 (C) 7.00%  $R^I = 10000 \cdot i = 1445.05 - 795.05 = 650$   
 (D) 7.25%  $\Rightarrow i = .065$   
 (E) 7.50%

3. A 10-year loan of 5000 is repaid with annual payments. The interest rate charged on the loan is 6% annual effective. Each annual payment equals 500 plus the interest due on the unpaid balance. When received, the payments are invested in an account that pays 5% annual effective. Determine the accumulated value at the end of the 10 years of the account in which the payments are invested.

(A) 6367  $R_1 = 500 + .06(5000) = 500 + 300$   
 $R_2 = 500 + .06(4500) = 500 + 270$   
 $\vdots$



- (D) 8039  
 (E) 8515

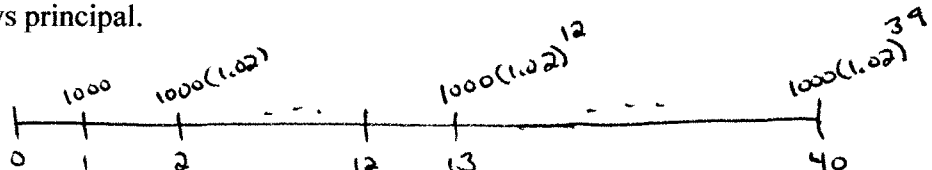
AV @ 5% aeir

$$AV = 500 S_{\overline{10}|.05} + 30(DS)_{\overline{10}|.05}$$

$$= 500 S_{\overline{10}|.05} + 30 \frac{10(1.05)^{10} - 5\overline{10}|.05}{.05} = 8515.58$$

4. A 20-year loan is amortized with semiannual payments, with the first payment equal to 1000 and each subsequent payment being 2% greater than the preceding one. Using an annual effective interest rate of 4.04%, determine the amount of the 13<sup>th</sup> payment that repays principal.

- (A) 570  
 (B) 610  
 (C) 640  
 (D) 670  
 (E) 700



$.0404 = aeir$   
 $\Rightarrow i = .02 = seir$

$B_{12} = 1000(1.02)^{12}v + 1000(1.02)^{13}v^2 + \dots (28 \text{ terms})$

$v = \frac{1}{1.02}$

$$\therefore B_{12} = 1000(1.02)^{12} [1 + v + \dots + v^{28}]$$

$$= 28000(1.02)^{12} = 34814.48$$

$$I_{13} = .02 B_{12} = 696.29$$

$$P_{13} = R_{13} - I_{13} = 1000(1.02)^{12} - 696.29 = 571.95$$

5. A 250,000 mortgage is amortized for 30 years with monthly payments using a 4.8% compounded monthly interest rate. Determine the number of the payment for which the interest payment is less than twice the principal repaid for the first time.  $i = .004 = meir$

(A) 84

$$I_k < 2P_k$$

(B) 85

$$I_k = R(1 - v^{361-k}) \quad P_k = Rv^{361-k}$$

(C) 86

$$\therefore R(1 - v^{361-k}) < 2Rv^{361-k} \Rightarrow v^{361-k} > \frac{1}{3}$$

(D) 87

$$(1.004)^{361-k} < 3 \Rightarrow k > 361 - \frac{\ln(3)}{\ln(1.004)} = 85.79+$$

(E) 88

$$\therefore k = 86$$

6. Two \$1000 par value bonds redeemable at par at the end of the same period are bought to yield 4% convertible quarterly. One bond costs \$1098 and has a coupon rate of 5% payable quarterly. The other bond has a coupon rate of 3.2% payable quarterly. Find the price of the second bond.

(A) 892

(B) 902

$$1098 = 12.5a_{\overline{n}|.01} + 1000v_{.01}^n$$

(C) 912

$$\Rightarrow n = 50$$

(D) 922

(E) 932

$$P = 8a_{\overline{n}|.01} + 1000v_{.01}^n = 921.6$$



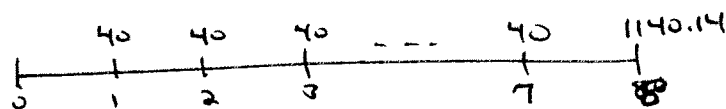
7. A 10-year 1000 face value bond with 4% annual coupons and a redemption value of 1200 is bought to yield 4% annual effective. When received, each coupon is invested in an account that pays 5% annual effective. Immediately prior to receiving the 8<sup>th</sup> coupon, the investor sells the bond to a new buyer at a price to yield the new buyer 8%. Determine the annual yield rate the original investor received during the 8 years of ownership.

- (A) 3.04%
- (B) 3.39%
- (C) 3.57%
- (D) 3.96%
- (E) 4.27%

Original Investor pays  $P = 40 a_{\overline{10}|.04} + 1200 v_{.04}^{10} = 1135.11$

New Buyer pays  $P = 40 + 40 a_{\overline{2}|.08} + 1200 v_{.08}^2 = 1140.14$

For 8 years of ownership by original investor, we have



$AV = 40 \ddot{s}_{\overline{7}|.05} + 1140.14 = 1482.10$  (AV using 5% acc)

$\therefore 1135.11 (1+i)^8 = 1482.10 \implies i \doteq 3.39\%$

8. For a 5-year bond with semiannual coupons bought to yield 4% compounded semiannually, the amount of interest earned during the 4<sup>th</sup> installment is 5 more than the amount of the coupon. Determine which statement is true.

- (A) The bond is bought at a premium of 24.52
- (B) The bond is bought at a discount of 24.52
- (C) The bond is bought at a premium of 51.59
- (D) The bond is bought at a discount of 51.59
- (E) None of the above

$I_4 = 5 + Fr$

$P_4 = Fr - I_4 = -5$

Since  $P_4 < 0$ , bond is bought at a discount.

$P_1 = P_4 v^3 = -5 (1.02)^{-3}$

Amount of Discount =  $-\sum_{k=1}^3 P_k$  (geometric) =  $-P_1 (1 + (1.02) + \dots + (1.02)^2)$  (10 terms)

$= 5 (1.02)^{-3} \cdot S_{\overline{3}|.02} = 51.59$

The bond is bought at a discount of 51.59.

9. A 30-year loan of 100,000 is amortized with level annual payments using an annual effective interest rate of 6%. Determine the total amount of interest paid on the loan during the middle 10 year period from time 10 years to time 20 years after loan inception.

(A) 39,200

(B) 40,100

(C) 41,000

(D) 41,900

(E) 42,800

$$R a_{\overline{30}|.06} = 100000 \Rightarrow R = 7264.89$$

$$\sum_{k=11}^{20} P_k = B_{10} - B_{20} = 83327.73 - 53470.23 = 29857.50$$

$$\sum_{k=11}^{20} I_k = 10R - \sum_{k=11}^{20} P_k = 42791.40$$

10. A 1000 face value bond with 6% annual coupons is bought at par when bought to yield 5% annual effective. Determine the redemption value.

(A) 1000

(B) 1050

(C) 1100

(D) 1150

(E) 1200

$$\text{Bought at Par} \Rightarrow P = C$$

$$P = C + (Fr - Ci) a_{\overline{n}|}$$

$$P = C \Rightarrow Fr = Ci$$

$$\therefore 60 = .05C$$

$$\Rightarrow C = 1200$$