

Each problem is worth 10 points.

1. A loan of 600,000 is being repaid using the sinking fund method in which the lender receives an annual effective interest rate of 5% and the sinking fund earns an annual effective rate of  $j$  for the first 10 years and 7% thereafter. Payments of \$45,040 are made at the end of each year for 20 years. These payments consist of the interest payment to the lender plus the sinking fund deposit. Calculate  $j$ .

- (A) 5.75%
- (B) 5.88%
- (C) 6.00%
- (D) 6.12%
- (E) 6.25%

2. Troy buys a ten-year 100 par value  $r\%$  bond with semi-annual coupons, redeemable at par. The price is 93.20 and assumes a nominal yield of 8% convertible semiannually. Determine  $r$ .

- (A) 8.0
- (B) 7.0
- (C) 6.0
- (D) 3.5
- (E) 3.0

3. Trapper Joe pays 1366 to purchase a 1000 face value 20-year callable bond with 8% semiannual coupons redeemable at 1440 on any coupon date starting at the end of year 15. Determine the minimum yield rate, as a nominal rate compounded semiannually, that Trapper Joe will earn on the bond.

- (A) 6.00%
- (B) 6.08%
- (C) 6.25%
- (D) 6.43%
- (E) 6.51%

4. A 30-year 1000 par value bond with 5% annual coupons has an amortized value of 1210 just after the 3<sup>rd</sup> coupon is paid and an amortized value of 1185 just after the 8<sup>th</sup> coupon is paid. Determine the redemption value of the bond.

- (A) Less than or equal to 1010
- (B) Greater than 1010, but less than or equal to 1020
- (C) Greater than 1020, but less than or equal to 1030
- (D) Greater than 1030, but less than or equal to 1040
- (E) Greater than 1040

5. A loan of 100,000 is repaid with payments at the end of each year for 30 years. Each of the first 15 payments equals 125% of the amount of interest then due. Each of the remaining payments is  $X$ . Interest is charged at an annual effective rate of 8%. Calculate  $X$ .

- (A) 8624
- (B) 8629
- (C) 8632
- (D) 8636
- (E) 8641

6. The following information is given regarding two bonds. Both bonds have annual coupons, both have coupon rate  $r$  and both are priced at effective annual rate of interest  $j$ .

Bond 1 has face amount 1000, matures in  $n$  years at face value, and has price equal to 807.47.

Bond 2 has face amount 2000, matures in  $2n$ -years at face value, and has price equal to 1452.29.

Find the price of a bond based on effective annual yield rate  $j$ , face amount 1000, coupon rate  $1.5r$ , and maturing in  $3n$  years at face value.

- (A) 850
- (B) 925
- (C) 1000
- (D) 1075
- (E) 1150

7. Willie purchases an  $n$ -year 100 par value bond with 6% annual coupons at a semi-annual effective yield rate of  $i$ . The book value of the bond at the end of year 6 (immediately after the coupon is paid) is 121.13, and the book value at the end of year 8 (immediately after the coupon is paid) is 118.77. Calculate the purchase price of the bond to the nearest dollar.

- (A) 122
- (B) 123
- (C) 125
- (D) 127
- (E) 128

8. A loan is to be repaid by  $2n$  level annual payments, starting one year after the loan is made. Just after the  $n$ -th payment, the borrower finds he still owes  $3/5$  of the original loan amount. What proportion of the following year's payment will represent principal?

- (A) 0.33
- (B) 0.50
- (C) 0.67
- (D) 0.75
- (E) 0.80

9. A homeowner borrows \$250,000 to be repaid over a 30-year period with level monthly payments beginning one month after the loan is made. With the payment at the end of the  $n^{\text{th}}$  month, the homeowner pays an extra 20,000. The homeowner continues to make the same monthly payments as before, but because of the extra payment the loan is now scheduled to be completely paid off with 138 more payments. The interest rate on the loan is a nominal annual rate of 12% compounded monthly. Determine  $n$ .

- (A) 165
- (B) 170
- (C) 175
- (D) 180
- (E) 185

10. A loan is being amortized by means of 120 level monthly payments at an annual effective interest rate of 10%. The amount of principal repaid in the 13<sup>th</sup> payment is 52.91. Determine the amount of each payment.

- (A) 110
- (B) 115
- (C) 120
- (D) 125
- (E) 130

MAP 4170  
Test 3Name \_\_\_\_\_  
Date: July 20, 2011

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A loan of 600,000 is being repaid using the sinking fund method in which the lender receives an annual effective interest rate of 5% and the sinking fund earns an annual effective rate of  $j$  for the first 10 years and 7% thereafter. Payments of \$45,040 are made at the end of each year for 20 years. These payments consist of the interest payment to the lender plus the sinking fund deposit. Calculate  $j$ .

(A) 5.75%

$$R_I = 600000(.05) = 30000 \Rightarrow R_{SF} = 15040$$

(B) 5.88%

(C) 6.00%

 (D) 6.12%

$$600000 = 15040 S_{\overline{10}|j} (1.07)^{10} + 15040 S_{\overline{10}|.07}$$

(E) 6.25%

$$\Rightarrow j \doteq 6.12\%$$

2. Troy buys a ten-year 100 par value  $r\%$  bond with semi-annual coupons, redeemable at par. The price is 93.20 and assumes a nominal yield of 8% convertible semiannually. Determine  $r$ .

(A) 8.0

 (B) 7.0

$$93.20 = 100 \left( \frac{.01r}{2} \right) a_{\overline{20}|.04} + 100 v_{.04}^{20}$$

(C) 6.0

(D) 3.5

$$\Rightarrow r = 7$$

(E) 3.0

3. Trapper Joe pays 1366 to purchase a 1000 face value 20-year callable bond with 8% semiannual coupons redeemable at 1440 on any coupon date starting at the end of year 15. Determine the minimum yield rate, as a nominal rate compounded semiannually, that Trapper Joe will earn on the bond.

- (A) 6.00%
- (B) 6.08%
- (C) 6.25%
- (D) 6.43%
- (E) 6.51%

$n$	$P$	$i = seir$
30	1366	.0304
⋮		
40	1366	.0300

Min yield rate  
is  $i^{(2)} = .06$

$$P = 40a_{\overline{n}|i} + 1440v_i^n$$

4. A 30-year 1000 par value bond with 5% annual coupons has an amortized value of 1210 just after the 3<sup>rd</sup> coupon is paid and an amortized value of 1185 just after the 8<sup>th</sup> coupon is paid. Determine the redemption value of the bond.

- (A) Less than or equal to 1010
- (B) Greater than 1010, but less than or equal to 1020
- (C) Greater than 1020, but less than or equal to 1030
- (D) Greater than 1030, but less than or equal to 1040
- (E) Greater than 1040

$$1210 = 50a_{\overline{5}|i} + 1185v_i^5$$

$$\Rightarrow i = .03749$$

$$1210 = 50a_{\overline{27}|i} + Cv_i^{27}$$

$$\Rightarrow C = 1000$$

5. A loan of 100,000 is repaid with payments at the end of each year for 30 years. Each of the first 15 payments equals 125% of the amount of interest then due. Each of the remaining payments is  $X$ . Interest is charged at an annual effective rate of 8%. Calculate  $X$ .

- (A) 8624  $B_0 = 100\,000$   
 (B) 8629  $B_1 = B_0(1.08) - 1.25I_1 = 1.08B_0 - 1.25(.08)B_0 = .98B_0$   
 (C) 8632  $B_2 = B_1(1.08) - 1.25I_2 = 1.08B_1 - 1.25(.08)B_1 = .98B_1 = .98^2B_0$   
 $\vdots$   
 (D) 8636  $B_{15} = .98^{15}B_0 = 73856.91 = Xa_{\overline{15}|.08}$   
 (E) 8641

$$\Rightarrow X \doteq 8629$$

6. The following information is given regarding two bonds. Both bonds have annual coupons, both have coupon rate  $r$  and both are priced at effective annual rate of interest  $j$ .

Bond 1 has face amount 1000, matures in  $n$  years at face value, and has price equal to 807.47.

Bond 2 has face amount 2000, matures in  $2n$ -years at face value, and has price equal to 1452.29.

Find the price of a bond based on effective annual yield rate  $j$ , face amount 1000, coupon rate  $1.5r$ , and maturing in  $3n$  years at face value.

- (A) 850  $807.47 = 1000ra_{\overline{n}|} + 1000v^n$   $ra_{\overline{n}|} = .80747 - v^n$   
 (B) 925  $1452.29 = 2000ra_{\overline{2n}|} + 2000v^{2n}$   
 (C) 1000  $P = 1500ra_{\overline{3n}|} + 1000v^{3n}$   
 (D) 1075  $ra_{\overline{2n}|} = ra_{\overline{n}|}(1+v^n) = (.80747 - v^n)(1+v^n)$   
 (E) 1150  $= .80747 - .19253v^n - v^{2n}$

$$\therefore 1452.29 = 2000(.80747 - .19253v^n - v^{2n}) + 2000v^{2n}$$

$$1452.29 = 1614.94 - 385.06v^n \Rightarrow v^n \doteq 0.4224$$

$$\therefore ra_{\overline{n}|} = .80747 - .4224 = .38507$$

$$P = 1500ra_{\overline{3n}|} + 1000v^{3n} = 1500ra_{\overline{n}|}(1+v^n+v^{2n}) + 1000v^{3n}$$

$$= 1500(.38507)(1 + .4224 + (.4224)^2) + 1000(.4224)^3$$

$$\therefore P = 1000$$



7. Willie purchases an  $n$ -year 100 par value bond with 6% annual coupons at a semi-annual effective yield rate of  $i$ . The book value of the bond at the end of year 6 (immediately after the coupon is paid) is 121.13, and the book value at the end of year 8 (immediately after the coupon is paid) is 118.77. Calculate the purchase price of the bond to the nearest dollar.

(A) 122

(B) 123

(C) 125

(D) 127

(E) 128

$$121.13 = 6 a_{\overline{2}|j} + 118.77 v_j^2 \Rightarrow j = .04 \text{ aeir}$$

$$P = 6 a_{\overline{8}|j} + 118.77 v_j^8 = 127.18$$

8. A loan is to be repaid by  $2n$  level annual payments, starting one year after the loan is made. Just after the  $n$ -th payment, the borrower finds he still owes  $3/5$  of the original loan amount. What proportion of the following year's payment will represent principal?

(A) 0.33

(B) 0.50

(C) 0.67

(D) 0.75

(E) 0.80

$$L = R a_{\overline{2n}|}$$

$$B_n \stackrel{\text{Pro}}{=} R a_{\overline{n}|} = \frac{3}{5} \cdot R a_{\overline{2n}|}$$

$$\therefore a_{\overline{n}|} = \frac{3}{5} a_{\overline{2n}|} = \frac{3}{5} \cdot a_{\overline{n}|} (1+v^n)$$

$$\Rightarrow 1 = \frac{3}{5} (1+v^n) \Rightarrow v^n = \frac{2}{3}$$

$$P_{n+1} = R \cdot v^{2n-(n+1)+1} = R v^n = \frac{2}{3} R$$

$\therefore$  the proportion of the  $(n+1)^{\text{st}}$  payment that represents principal is  $\frac{2}{3}$

9. A homeowner borrows \$250,000 to be repaid over a 30-year period with level monthly payments beginning one month after the loan is made. With the payment at the end of the  $n^{\text{th}}$  month, the homeowner pays an extra 20,000. The homeowner continues to make the same monthly payments as before, but because of the extra payment the loan is now scheduled to be completely paid off with 138 more payments. The interest rate on the loan is a nominal annual rate of 12% compounded monthly. Determine  $n$ .

- (A) 165  
 (B) 170  
 (C) 175  
 (D) 180  
 (E) 185
- $250000 = R a_{\overline{360}|.01} \Rightarrow R = 2571.53$   
 Before the extra payment,  $B_n = R a_{\overline{360-n}|.01}$   
 $\therefore B_n - 20000 = R a_{\overline{138}|.01}$   
 $\Rightarrow 2571.53 a_{\overline{360-n}|.01} = 2571.53 a_{\overline{138}|.01} + 20000$   
 $\Rightarrow 360 - n = 175$   
 $\Rightarrow n = 185$

10. A loan is being amortized by means of 120 level monthly payments at an annual effective interest rate of 10%. The amount of principal repaid in the 13<sup>th</sup> payment is 52.91. Determine the amount of each payment.

- (A) 110  
 (B) 115  
 (C) 120  
 (D) 125  
 (E) 130
- $P_{13} = 52.91 = R v_j^{120-13+1} = R v_j^{108}$   
 $j = \text{meir} \Rightarrow \text{---} =$   
 $(1+j)^{12} = 1.1 \Rightarrow v_j^{12} = v_{.1}$   
 $R = 52.91 (1+j)^{108} = 52.91 (1.1)^9 = 124.76$