Extra Problems for Test 2

1. A person deposits 100 at the beginning of each year for 20 years. Simple interest at an annual rate of i is credited to each deposit from the date of deposit to the end of the twenty year period. The total amount thus accumulated is 2840. If instead, compound interest had been credited at an effective annual rate of i, what would the accumulated value of these deposits have been at the end of twenty years?

(A) 2980   (B) 3100   (C) 3200   (D) 3310   (E) 3470

2. A perpetuity pays 1 at the end of every year plus an additional 1 at the end of every second year. The present value of the perpetuity is K for $i \geq 0$. Determine K.

(A) $\dfrac{i+3}{i(i+2)}$ (B) $\dfrac{i+2}{i(i+1)}$ (C) $\dfrac{i+1}{i^2}$ (D) $\dfrac{3}{2i}$ (E) $\dfrac{i+1}{i(i+2)}$

3. An annuity pays 1 at the end of each 4-year period for 40 years. Given $a_{\overline{8}|i} = k$, find the present value of the annuity.

(A) $\dfrac{1-(1-ik)^5}{1-(1-ik)^5}$ (B) $\dfrac{1-(1-ik)^{40}}{1-(1-ik)^5}$ (C) $\dfrac{1-(1-ik)^5}{i}$

(D) $\dfrac{1-(1-ik)^{40}}{(1-ik)^{-.5}-1}$ (E) $\dfrac{1-(1-ik)^5}{(1-ik)^{-.5}-1}$

4. You are given $\delta_t = \dfrac{2}{10+t}$, $t \geq 0$. Calculate $a_{\overline{4}|}$.

(A) 2.34   (B) 2.62   (C) 2.85   (D) 3.01   (E) 3.23

5. You are given two series of payments. Series A is a perpetuity with payments of 1 at the end of each of the first 2 years, 2 at the end of each of the next 2 years, 3 at the end of each of the next 2 years, and so on. Series B is a perpetuity with payments of K at the end of each of the first 3 years, $2K$ at the end of the next 3 years, $3K$ at the end of each of the next 3 years, and so on. The present values of the two series of payments are equal. Calculate K.

(A) $\dfrac{3i}{2}$ (B) $\dfrac{3d}{2}$ (C) $\dfrac{a_{\overline{3}|}}{a_{\overline{2}|}}$ (D) $\dfrac{a_{\overline{3}|}}{\ddot{a}_{\overline{2}|}}$ (E) $\dfrac{\ddot{s}_{\overline{3}|}}{\ddot{s}_{\overline{2}|}}$

Extra Problems for Test 2

6. You are given an annuity-immediate with 11 annual payments of 100 and a final larger payment at the end of 12 years. At an annual effective interest rate of 3.5%, the present value at time 0 of all payments is 1000.

Using an annual effective interest rate of 1%, calculate the present value at the beginning of the ninth year of all remaining payments.

   (A) 412   (B) 419   (C) 432   (D) 439   (E) 446

7. The proceeds of a 10,000 death benefit are left on deposit with an insurance company for seven years at an effective annual interest rate of 5%.

The balance at the end of seven years is paid to the beneficiary in 120 equal monthly payments of X, with the first payment made immediately. During the payout period, interest is credited at an annual effective rate of 3%.

Calculate X.

   (A) 117   (B) 118   (C) 129   (D) 135   (E) 158

8.   You are given:

(i) The present value of an annuity-due that pays 300 every 6 months during the first 15 years and 200 every 6 months during the second 15 years is 6000.

(ii) The present value of a 15-year deferred annuity-due that pays 350 every 6 months for 15 years is 1580.

(iii) The present value of an annuity-due that pays 100 every 6 months during the first 15 years and 200 every 6 months during the next 15 years is X.

The same interest rate is used in all calculations. Determine X.

   (A) 2302   (B) 2402   (C) 2502   (D) 2602   (E) 2702

9. At a nominal rate of interest i, convertible semiannually, the present value of a series of payments of 1 at the end of every 2 years forever, is 5.89. Calculate i.

   (A) 6%   (B) 7%   (C) 8%   (D) 9%   (E) 10%

Extra Problems for Test 2

10. You are given $\delta_t = \dfrac{1}{1+t}$, for $0 \le t \le 5$. Calculate $s_{\overline{5}|}$.

(A) 7.5   (B) 8.7   (C) 10.5   (D) 13.7   (E) 16.0

11. John deposits 100 at the end of each year for 20 years into a fund earning an annual effective interest rate of 7%.

Mary makes 20 deposits into a fund at the end of each year for 20 years. The first 10 deposits are 100 each, while the last 10 deposits are $100 + X$ each. The fund earns an annual effective interest rate of 8% during the first 10 years and 6% annual effective interest thereafter.

At the end of 20 years, the amount in John's fund equals the amount in Mary's fund.

Calculate X.

(A) 8   (B) 10   (C) 12   (D) 14   (E) 16

12. Seth, Janice, and Lori each borrow 5000 for five years at a nominal interest rate of 12%, compounded semiannually. Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years. Janice pays interest at the end of every six month period as it accrues and the principal at the end of five years. Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

(A) 8718   (B) 8728   (C) 8738   (D) 8748   (E) 8758

Extra Problems for Test 2

13. A 10-year loan of 2000 is to be repaid with payments at the end of each year.

It can be repaid under the following two options:

(i) Equal annual payments at an annual effective rate of 8.07%.

(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective rate of i.

The sum of the payments under option (i) equals the sum of the payment under option (ii)

Determine i.

   (A) 8.75%   (B) 9.00%   (C) 9.25%   (D) 9.50%   (E) 9.75%

14. A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of X to a fund for 25 years. The fund earns a nominal rate of interest of 8% compounded monthly.

Each 1000 will provide for 9.65 of income at the beginning of each month starting on his 65th birthday until the end of his life.

Calculate X.

   (A) 324.73   (B) 326.89   (C) 328.12   (D) 355.45   (E) 450.65

15. At an annual effective interest rate of i, $i > 0$, both of the following annuities have a present value of X.

(i) A 20-year annuity-immediate with annual payments of 55

(ii) A 30-year annuity immediate with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years.

Calculate X.

   (A) 575   (B) 585   (C) 595   (D) 605   (E) 615

Extra Problems for Test 2

16. You are given a perpetual annuity-immediate with annual payments increasing in geometric progression, with a common ratio of 1.07. The annual effective interest rate is 12%. The first payment is 1. Calculate the present value of this annuity.

   (A) 18   (B) 19   (C) 20   (D) 21   (E) 22

17. An annuity-immediate pays an initial benefit of 1 per year, increasing by 10.25% every four years. The annuity is payable for 40 years. Using an annual effective interest rate of 5%, determine an expression for the present value of this annuity.

   (A) $\left(1+v^2\right)\cdot \ddot{a}_{\overline{20}|}$ (B) $\left(1+v^2\right)\cdot a_{\overline{20}|}$ (C) $2\cdot a_{\overline{20}|}$ (D) $\dfrac{a_{\overline{20}|}}{s_{\overline{2}|}}$ (E) $\dfrac{a_{\overline{40}|}}{a_{\overline{2}|}}$

18. Chris makes annual deposits into a bank account at the beginning of each year for 20 years. Chris' initial deposit is equal to 100, with each subsequent deposit k% greater than the previous year's deposit. The bank credits interest at an annual effective interest rate of 5%.

At the end of 20 years, the accumulated value in Chris' account is equal to 7276.35.

Given $k > 5$, calculate k.

   (A) 8.06   (B) 8.21   (C) 8.36   (D) 8.51   (E) 8.68

19. Barbara purchases an increasing perpetuity with payments occurring at the end of every 2 years. The first payment is 1, the second one is 2, the third one is 3, and so on. The price of the perpetuity is 110. Calculate the annual effective interest rate.

   (A) 4.50%   (B) 4.62%   (C) 4.75%   (D) 4.88%   (E) 5.00%

Extra Problems for Test 2

20. Joe can purchase one of two annuities:

Annuity 1: A 10-year decreasing annuity-immediate, with annual payments 10, 9, …, 1.

Annuity 2: A perpetuity-immediate with annual payments. The perpetuity pays 1 in year 1, 2 in year 2, …, and 11 in year 11. After year 11, the payments remain constant at 11.

At an annual effective interest rate of i, the present value of Annuity 2 is twice the present value of Annuity 1.

Calculate the present value of Annuity 1.

   (A) 36.4   (B) 37.4   (C) 38.4   (D) 39.4   (E) 40.4

21. Olga buys a 5-year increasing annuity for X.

Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2.

The nominal interest rate is 9% convertible quarterly.

Calculate X.

   (A) 2680   (B) 2730   (C) 2780   (D) 2830   (E) 2880

22. A varying immediate annuity with a term of $2n$ years has a first payment equal to 1. Thereafter payments increase by 1 each year until they reach n at the end of n years. Payments remain at n for year $n+1$, and then decrease by 1 each year, with a final payment of 1 at the end of $2n$ years. Derive an expression for the present value of this annuity.

   (A) $a_{\overline{n}|}\left(\dfrac{1}{d}-\dfrac{v^n}{i}\right)$ (B) $a_{\overline{n}|}\left(\dfrac{1}{d}+\dfrac{v^n}{i}\right)$ (C) $a_{\overline{n}|}\left(\dfrac{1}{d}-\dfrac{v^n}{d}\right)$

   (D) $a_{\overline{n}|}\left(\dfrac{1}{d}+\dfrac{v^n}{d}\right)$ (E) $a_{\overline{n}|}\left(\dfrac{1}{i}+\dfrac{v^n}{d}\right)$

Extra Problems for Test 2

23.   An 11-year annuity has a series of payments 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, with the first payment made at the end of the second year. The present value of this annuity is 25 at interest rate i.

A 12-year annuity has a series of payments 1, 2, 3, 4, 5, 6, 6, 5, 4, 3, 2, 1, with the first payment made at the end of the first year.

Calculate the present value of the 12-year annuity at interest rate i.

   (A) 29.5   (B) 30.0   (C) 30.5   (D) 31.0   (E) 31.5

24.   Mary purchases an increasing annuity-immediate for 50,000 that makes twenty annual payments as follows:

(i) P, $2P$, …, $10P$ in years 1 through 10; and

(ii) $10(1.05)P$, $10(1.05^2)P$, …, $10(1.05^{10})P$ in years 11 through 20.

The annual effective interest rate is 7% for the first 10 years, and 5% thereafter.

Calculate P.

   (A) 564   (B) 574   (C) 584   (D) 594   (E) 604

25.   Joan has won a lottery that pays 1000 per month in the first year, 1100 per month in the second year, 1200 per month in the third year, and so on. Payments are made at the end of each month for 10 years.

Using an effective interest rate of 3% per annum, calculate the present value of this prize.

   (A) 107,000   (B) 114,000 (C) 123,000 (D) 135,000 (E) 148,000

26.   You are given $\int_0^n \overline{a}_{\overline{t}|}\,dt = 100$. Calculate $\overline{a}_{\overline{n}|}$.

   (A) $100n\delta$ (B) $n\delta$ (C) $n - 100\delta$ (D) $100 - n\delta$ (E) $n - \dfrac{\delta}{100}$

Extra Problems for Test 2

27. Payments are made to an account at a continuous rate of $(8k + tk)$, where $0 \le t \le 10$.

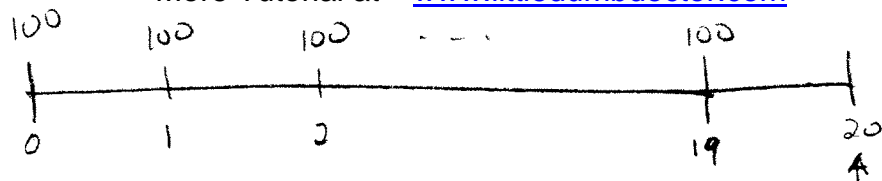Interest is credited at a force of interest of $\delta_t = \dfrac{1}{8+t}$.

After 10 years, the account is worth 20,000.

Calculate k.

    (A) 111   (B) 116   (C) 121   (D) 126   (E) 131

1)

$$\begin{array}{c|ccccc} & 100 & 100 & 100 & \cdots & 100 & \\ \hline 0 & 1 & 2 & & 19 & 20 \end{array}$$

Simple Interest: $AV = 2840 = 100(1+20i)$
$\qquad\qquad\qquad\qquad\qquad + 100(1+19i)$

$\qquad\qquad\qquad\qquad + \cdots$
$\qquad\qquad\qquad + 100(1+i)$

$$\begin{array}{r} 100 + 2000i \\ 100 + 1900i \\ \vdots \\ 100 + 100i \\ \hline 20(100) + i(\cdots) \end{array}$$

$\therefore\ 2840 = \underbrace{20(100)}_{2000} + 100i\ \underbrace{(1+2+\cdots+20)}_{\frac{20(21)}{2} = 210}$

$\qquad\qquad\qquad\qquad\qquad\qquad 1+2+\cdots+n = \frac{n(n+1)}{2}$

$$\therefore\ i = \frac{8.4}{210} = .04$$

Compound Interest: $AV_{20} = 100\ \ddot{s}_{\overline{20}|.04} = 3096.92$

$\mathcal{C}alculator$: BGN mode since annuity due

$\qquad$ 2nd PMT 2nd Enter 2nd CPT

$\boxed{BGN}$ 20 $\boxed{N}$ 4 $\boxed{I/Y}$ 0 $\boxed{PV}$ 100 $\boxed{+/-}$ $\boxed{PMT}$

$\qquad\qquad$ $\boxed{CPT}$ $\boxed{FV}$ = 3096.92

2)

$$\begin{array}{c|ccccc} & & & & \cdots & \\ \hline \end{array}$$
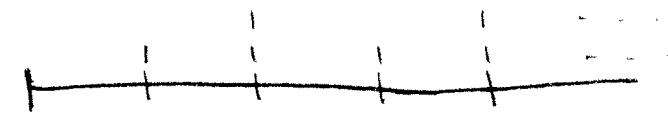
$PV = K = \dfrac{1}{i} + \dfrac{1}{j}$

$i = $ a eir
$j = $ 2-year eir $= (1+i)^2 - 1$
$\qquad\qquad\qquad = 2i + i^2$
$\qquad\qquad\qquad = i(2+i)$

$\therefore\ K = \dfrac{1}{i} + \dfrac{1}{i(2+i)} = \dfrac{3+i}{i(2+i)}$

3) $\qquad a_{\overline{8}|i} = k \qquad\qquad i = aeir \qquad\qquad \Big\} \begin{array}{l}(1+i)^4 = 1+j \\ v_i^4 = v_j\end{array}$

$PV = a_{\overline{10}|j} \qquad\qquad\qquad j = 4\text{-year } eir$

$$= \frac{1 - v_j^{10}}{j} = \frac{1 - v_i^{40}}{(1+i)^4 - 1}$$

Since $a_{\overline{8}|i} = k$, then $\quad 1 - v_i^8 = ki$

$$\implies v_i^8 = 1 - ik \qquad\qquad v_i^{40} = (1-ik)^5$$

$$(1+i)^{-8} = 1 - ik$$

$$\implies (1+i)^4 = (1-ik)^{-.5}$$

$$\therefore PV = \frac{1 - (1-ik)^5}{(1-ik)^{-.5} - 1}$$

4)
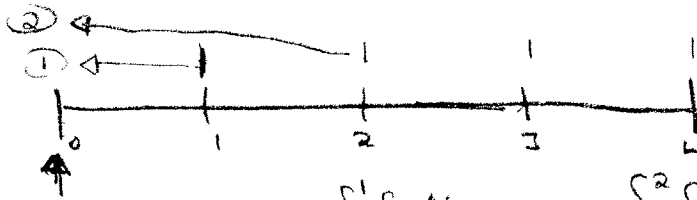
$$PV = a_{\overline{4}|} = \underbrace{e^{-\int_0^1 \delta_t\, dt}}_{=\,①} + \underbrace{e^{-\int_0^2 \delta_t\, dt}}_{=\,②} + e^{-\int_0^3 \delta_t\, dt} + e^{-\int_0^4 \delta_t\, dt}$$
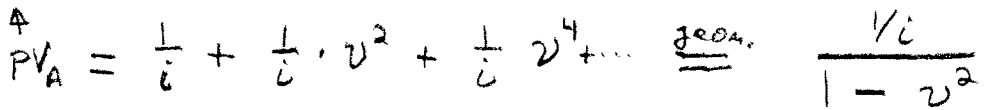
$$e^{-\int_0^n \delta_t\, dt} = e^{-\int_0^n \frac{2}{10+t}\, dt} = e^{-2\ln(10+t)\big|_0^n}$$

$$= e^{-2\ln\left(\frac{10+n}{10}\right)} = \left(\frac{10}{10+n}\right)^2$$

$$\therefore PV = \left(\frac{10}{11}\right)^2 + \left(\frac{10}{12}\right)^2 + \left(\frac{10}{13}\right)^2 + \left(\frac{10}{14}\right)^2$$

5) A:

$$PV_A = \frac{1}{i} + \frac{1}{i}\cdot v^2 + \frac{1}{i}v^4 + \cdots \quad \underset{=}{geom.} \quad \frac{1/i}{1-v^2}$$

B:
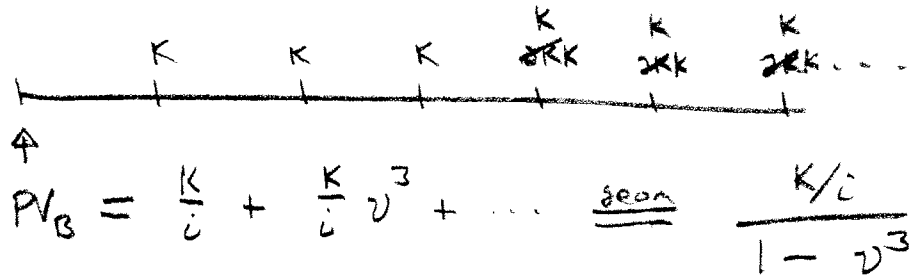
$$PV_B = \frac{K}{i} + \frac{K}{i}v^3 + \cdots \quad \underset{=}{geom} \quad \frac{K/i}{1-v^3}$$

$$\Rightarrow K = \frac{1-v^3}{1-v^2} = \frac{a_{\overline{3}|}}{a_{\overline{2}|}} \quad \left(Note: \frac{\ddot{a}_{\overline{3}|}}{\ddot{a}_{\overline{2}|}} = \frac{a_{\overline{3}|}}{a_{\overline{2}|}}\right)$$

6)
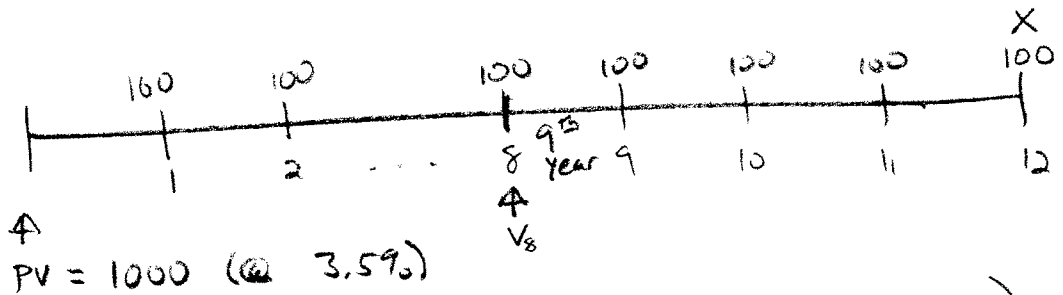
$$PV = 1000 \;(@ \; 3.5\%)$$

$$1000 = 100\, a_{\overline{12}|} + K v^{12} \quad (K = X + 100)$$

OR! $$1000 = 100\, a_{\overline{12}|3.5\%} + X v^{12}_{.035} \implies X = FV = 50.87$$

END   12 [N] 3.5 [I/Y] 1000 [PV] 100 [+/-] [PMT] [CPT] [FV]

$$V_8 = 100\, a_{\overline{4}|.01} + X v^{4}_{.0} = 439$$

Continuing with calculator:

4 [N] 1 [I/Y] [CPT] [PV] = 439

7)

(X)

$10000 \longrightarrow 10000(1.05)^7$ (X) (X) (X) -- 120 monthly payments

$7yrs$
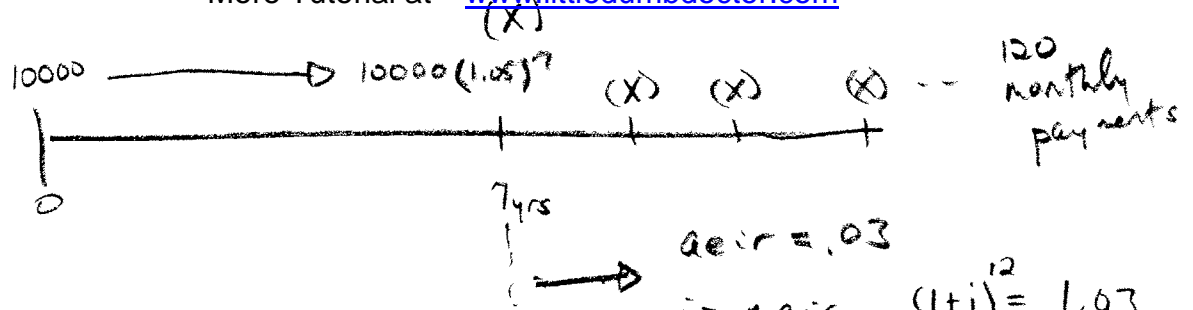
$aeir = .03$

$j = meir$ $(1+j)^{12} = 1.03$

$Get \; j = (1.03)^{1/12} - 1 = .2466\%$

$$10000(1.05)^7 = X \, \ddot{a}_{\overline{120}|j}$$

PV

$\left. \begin{array}{l} n = 120 \\ j = .2466\% \\ FV = 0 \end{array} \right\}$ $\boxed{CPT} \; \boxed{PMT} = 135.6$

8)(i) $6000 = 300 \, \ddot{a}_{\overline{30}|} + 200 \, \ddot{a}_{\overline{30}|} \cdot v^{30}$ (seir)

(ii) $1580 = 350 \, \ddot{a}_{\overline{30}|} \cdot v^{30}$ (seir)

(iii) $X = 100 \, \ddot{a}_{\overline{30}|} + \underbrace{200 \, \ddot{a}_{\overline{30}|} \cdot v^{30}}$ (seir)

$= \frac{1580}{350}(200)$ by (ii)

(i) $\Longrightarrow$ $6000 = 300 \, \ddot{a}_{\overline{30}|} + \frac{1580}{350}(200)$

$\Longrightarrow$ $300 \, \ddot{a}_{\overline{30}|} = 6000 - \frac{1580}{350}(200)$

$\therefore X = \frac{1}{3}\left[6000 - \frac{1580}{350}(200) \right] + \frac{1580}{350}(200)$

$\doteq 2602$

9)

$PV = \frac{1}{j} = 5.89$ $j = 2\text{-year} \quad eir$
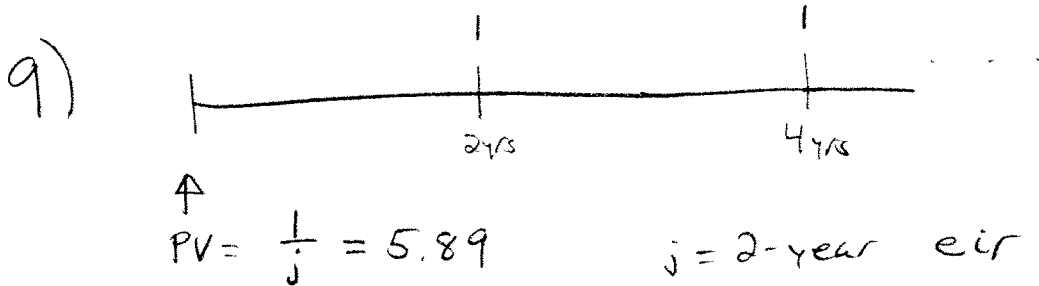
$i = i^{(2)}$

$\left(1 + \frac{i^{(2)}}{2}\right)^4 = 1 + j \implies i^{(2)} = 2\left[\left(1 + \frac{1}{5.89}\right)^{1/4} - 1\right] = .08$

10)
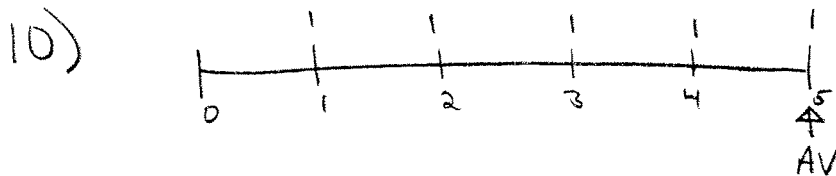
$$AV = e^{\int_1^5 \delta_t \, dt} + e^{\int_2^5 \delta_t \, dt} + e^{\int_3^5 \delta_t \, dt} + e^{\int_4^5 \delta_t \, dt} + e^{\int_5^5 \delta_t \, dt}$$

$$e^{\int_n^5 \delta_t \, dt} = e^{\int_n^5 \frac{1}{1+t} \, dt} = e^{\ln(1+t)|_n^5}$$

$$= e^{\ln(6) - \ln(1+n)} = e^{\ln\left(\frac{6}{1+n}\right)} = \frac{6}{1+n}$$

$$\therefore AV = \frac{6}{2} + \frac{6}{3} + \frac{6}{4} + \frac{6}{5} + \frac{6}{6}$$

$$= 8.7$$

11) J:
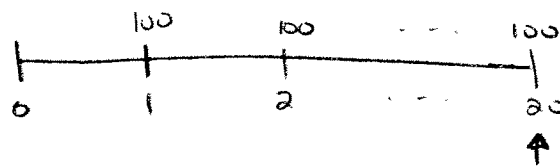
$aeir = 7\%$

$$AV_J = 100\ S_{\overline{20}|.07}$$

M:

$aeir = 8\%$ $aeir = 6\%$
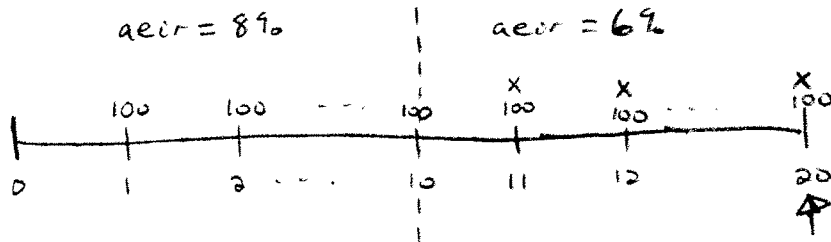
$$AV_M = 100\ S_{\overline{10}|.08}\ (1.06)^{10} + (100+X)\ S_{\overline{10}|.06}$$

$$AV_M = AV_J$$

$$\therefore\ \underbrace{100\ S_{\overline{10}|.08}\ (1.06)^{10}}_{PV} + \underbrace{(100+X)}\ S_{\overline{10}|.06} = \underbrace{100\ S_{\overline{20}|.07}}_{FV}$$

$$\underset{PMT}{}$$

$$100+X = 114.20 \implies X = 14.20$$

12) S: Interest $= 5000(1.06)^{10} - 5000 = 3954.24$

J: Interest $= 5000(.06) = 300$ every 6 months

for a total of $300(10) = 3000$

L: $5000 = R \cdot a_{\overline{10}|.06} \implies R = 679.34$ (PMT)

$\therefore$ Interest $= 679.34(10) - 5000 = 1793.40$

$$\sum = 3954.24 + 3000 + 1793.40 \doteq 8748$$

13) (i) $2000 = R\, a_{\overline{10}|8.07\%} \Rightarrow R = 299$

$$\Sigma_i = 10(299) = 2990$$

(ii)
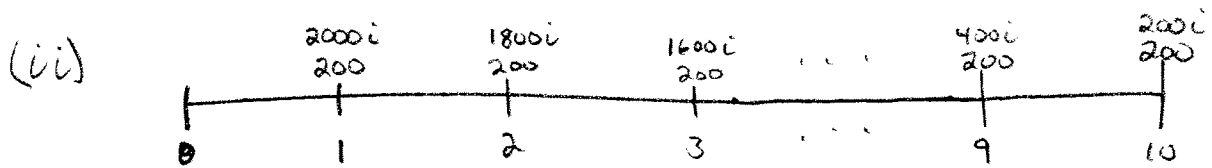
$$\Sigma_{ii} = 10(200) + 200i(1 + 2 + \cdots + 10)$$

$$= 2000 + 200i \cdot \frac{10(11)}{2} = 2000 + 11000i$$

$$\Sigma_i = \Sigma_{ii} \Rightarrow 2990 = 2000 + 11000i \Rightarrow i = .09$$

14)

$i^{(12)} = 8\%$

Pay X per month

Receive 3000/month for life

time 0
age 40

25
65
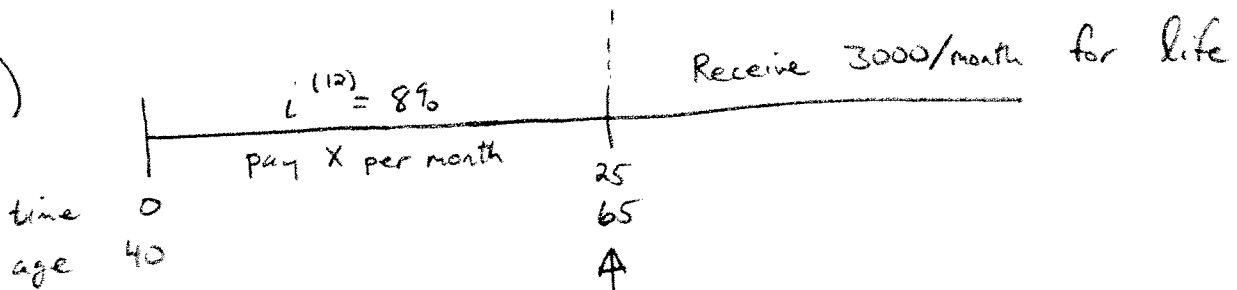
$AV = X\, \ddot{s}_{\overline{300}|.\overline{6}\%} = $ amount needed to pay 3000/month for life

$\frac{.08}{12} = .00\overline{6} = $ meir

1000 pays 9.65/month for life

$\Rightarrow \frac{1000}{9.65}$ pays 1/month for life

$\Rightarrow \frac{1000}{9.65}(3000)$ pays 3000/month for life

$\therefore X \cdot \ddot{s}_{\overline{300}|.\overline{6}\%} = \underbrace{\frac{1000}{9.65}(3000)}_{FV}$

$\boxed{BGN}$ 300 $\boxed{N}$.$\overline{6}$ $\boxed{I/Y}$ 0 $\boxed{PV}$ $\underbrace{\boxed{FV}\ \boxed{CPT}\ \boxed{PMT}}_{}$ $X = 324.72$

15) (i) $X = 55a_{\overline{20}|} = 55a_{\overline{10}|} + 55a_{\overline{10}|} \cdot v^{10}$

(ii) $X = 30a_{\overline{10}|} + 60a_{\overline{10}|}v^{10} + 90a_{\overline{10}|}v^{20}$ $\Bigg\} =$

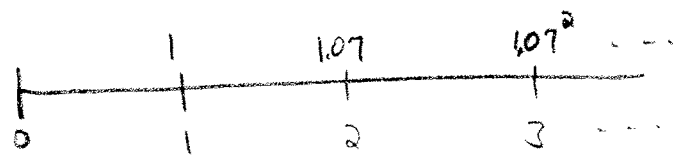$\therefore \quad 55 + 55v^{10} = 30 + 60v^{10} + 90v^{20}$

$$90v^{20} + 5v^{10} - 25 = 0$$

$$v^{10} = \frac{-5 \pm \sqrt{25 + 4(90)(25)}}{180} = \frac{1}{2}$$

$$\Rightarrow (1+i)^{10} = 2 \Rightarrow i = 2^{1/10} - 1$$

$$X = 55a_{\overline{20}|i} \quad \underline{\text{Calculator}} \quad 575$$

16)

$PV = v + 1.07v^2 + 1.07^2v^3 + \cdots \quad \underline{\text{geometric}} \quad \frac{v}{1 - 1.07v} \cdot \frac{1+i}{1+i} = \frac{1}{(1+i) - 1.07}$

$\underline{\underline{i = .12}} \quad \frac{1}{.05} = 20$

17)
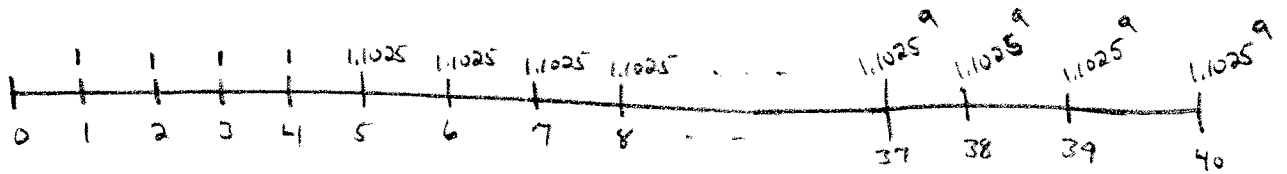
Replace the above stream by
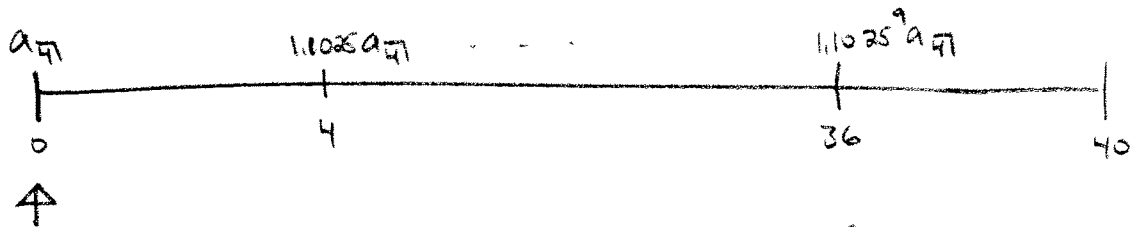
$$PV = a_{\overline{4}|} + 1.1025\, a_{\overline{4}|}\, v^4 + 1.1025^2\, a_{\overline{4}|}\, v^8 + \cdots + 1.1025^9\, a_{\overline{4}|}\, v^{36}$$

Note: $1.1025\, v^4 = (1.05)^2 \cdot v^4 = (1.05)^2 \left(\frac{1}{1.05}\right)^4 = (1.05)^{-2} = v^2$

$1.1025^2\, v^8 = (1.05)^4\, v^8 = (1.05)^4 \left(\frac{1}{1.05}\right)^8 = (1.05)^{-4} = v^4$

$\vdots$

$$\therefore PV = a_{\overline{4}|} + a_{\overline{4}|}\, v^2 + a_{\overline{4}|}\, v^4 + \cdots + a_{\overline{4}|}\, v^{18}$$

$$= a_{\overline{4}|}\left(1 + v^2 + v^4 + \cdots + v^{18}\right)$$

$$= a_{\overline{4}|} \cdot \frac{1 - v^{20}}{1 - v^2} = \frac{1 - v^4}{i} \cdot \frac{1 - v^{20}}{1 - v^2}$$

$$= \frac{1 - v^{20}}{i} \cdot \frac{1 - v^4}{1 - v^2} = a_{\overline{20}|} \cdot \frac{(1 + v^2)(1 - v^2)}{1 - v^2}$$

$$= a_{\overline{20}|}\left(1 + v^2\right)$$

18)
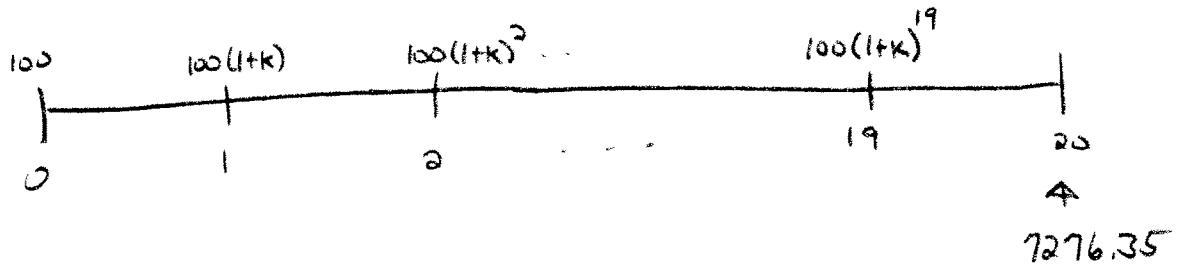
$$7276.35 = 100(1.05)^{20} + 100(1+k)(1.05)^{19} + \cdots + 100(1+k)^{19}(1.05)$$

$$\therefore \ 7276.35(1.05)^{-20} = 100 + 100\left(\frac{1+k}{1.05}\right) + \cdots + 100\left(\frac{1+k}{1.05}\right)^{19}$$

Since $k > 5\%$, $\frac{1+k}{1.05} > 1$.

Think of $\frac{1+k}{1.05} = 1+i$

$$\therefore \ 7276.35(1.05)^{-20} = 100 + 100(1+i) + \cdots + 100(1+i)^{19}$$

$$= 100 \ S_{\overline{20}|i}$$

Using calculator, $i = 3.2\%$

$$\therefore \ \frac{1+k}{1.05} = 1.032 \implies k = 8.36\%$$

19)

$$PV = 110 = \frac{P}{j} + \frac{Q}{j^2} \ \xRightarrow{P=Q=1} \ \frac{1}{j} + \frac{1}{j^2} \qquad j = 2\text{-year eir}$$

$$\therefore \ 110j^2 = j+1 \implies 110j^2 - j - 1 = 0$$

$$\implies j = \frac{1 \pm \sqrt{1 + 4(110)(1)}}{220} = 10\%$$

$$i = aeir \implies (1+i)^2 = 1+j = 1.1 \implies i = 4.88\%$$

20) Annuity 1: $PV_1 = (Da)_{\overline{10}|}$

Annuity 2:

$PV_2 = (Ia)_{\overline{10}|} + \frac{11}{i} \cdot v^{10}$

$PV_2 = 2\, PV_1$

$\Rightarrow (Ia)_{\overline{10}|} + \frac{11}{i} v^{10} = 2 \cdot (Da)_{\overline{10}|}$

$\left[\dfrac{\ddot{a}_{\overline{10}|} - 10 v^{10}}{i} + \dfrac{11 v^{10}}{i} = 2 \cdot \dfrac{10 - a_{\overline{10}|}}{i} \right] * i$
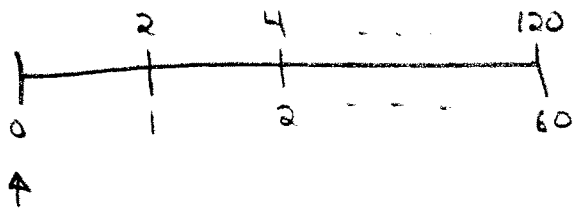
$\Rightarrow \underbrace{\ddot{a}_{\overline{10}|} + v^{10}}_{} = 20 - 2\, a_{\overline{10}|}$

$= \ddot{a}_{\overline{11}|} = 1 + a_{\overline{10}|}$

$\therefore \quad 1 + a_{\overline{10}|} = 20 - 2 a_{\overline{10}|}$

$\Rightarrow \quad a_{\overline{10}|} = \dfrac{19}{3} \xrightarrow{\text{calculator}} i = 9.3\%$

$\therefore \quad PV_1 = (Da)_{\overline{10}|} = \dfrac{10 - a_{\overline{10}|}}{i} = \dfrac{10 - \frac{19}{3}}{.093} = 39.4$
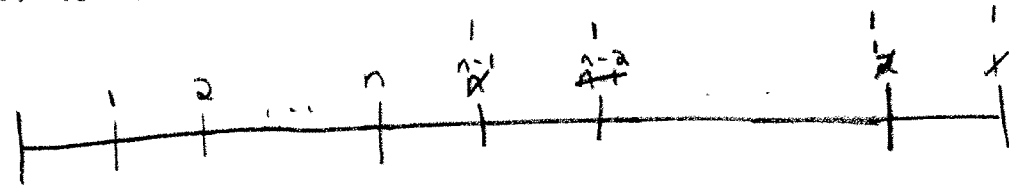
21)

$$PV = 2 \cdot (Ia)_{\overline{60}|j} \qquad j = meir$$

$$i^{(4)} = .09 \implies 1 + \frac{.09}{4} = (1+j)^3 \implies j = .00744$$

$$\therefore PV = 2 \; \frac{\ddot{a}_{\overline{60}|j} - 60v_j^{60}}{j} = 2730$$
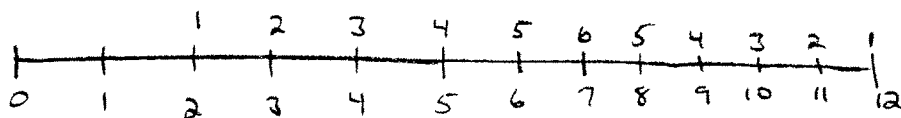
22) Not a rainbow

$$PV = PV \,(rainbow) + PV\,(n\text{-year deferred } n\text{-year annuimn})$$

$$= a_{\overline{n}|} \cdot \ddot{a}_{\overline{n}|} + a_{\overline{n}|} \cdot v^n$$

$$= a_{\overline{n}|} (\ddot{a}_{\overline{n}|} + v^n) = a_{\overline{n}|} \cdot \ddot{a}_{\overline{n+1}|} = a_{\overline{n}|} \left(\frac{1 - v^{n+1}}{d} \right)$$

$$= a_{\overline{n}|} \left(\frac{1}{d} - \frac{v \cdot v^n}{d} \right) \overset{d=iv}{=\!=} a_{\overline{n}|} \left(\frac{1}{d} - \frac{v^n}{i} \right)$$
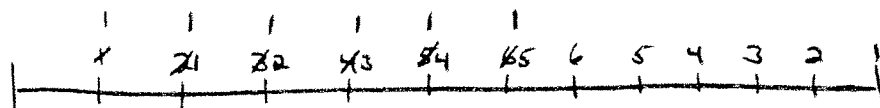
23) 11-year annuity :

$$PV = PV(\text{rainbow}) \cdot v = a_{\overline{6}|} \cdot \underbrace{\ddot{a}_{\overline{6}|} \cdot v}_{= a_{\overline{6}|}} = (a_{\overline{6}|})^2 = 25$$

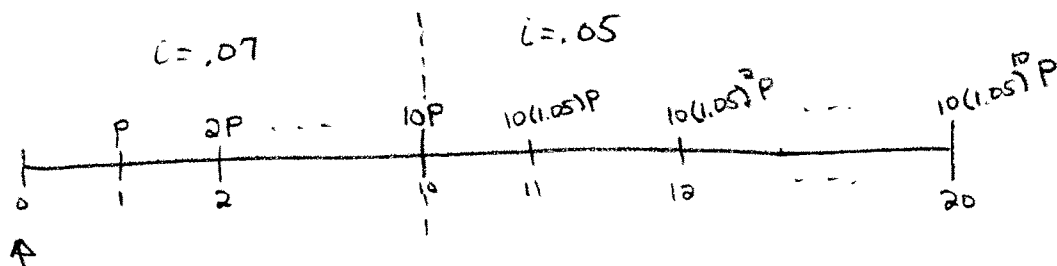$$\Rightarrow a_{\overline{6}|} = 5$$

12-year annuity :

$$PV = a_{\overline{6}|} + PV(\text{above 11-year annuity})$$

$$= 5 + 25 = 30$$

24)

$$50000 = P \cdot (Ia)_{\overline{10}|.07} + 10(1.05)P \cdot v_{.05} \cdot v_{.07}^{10}$$

$$+ 10(1.05)^2 P \cdot v_{.05}^2 \cdot v_{.07}^{10}$$

$$+ \cdots$$
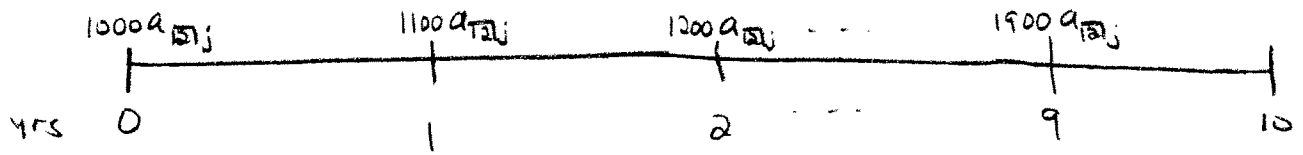
$$+ 10(1.05)^{10} P \cdot v_{.05}^{10} \cdot v_{.07}^{10}$$

$$50000 = P \cdot (Ia)_{\overline{10}|.07} + \underbrace{10P\, v_{.07}^{10} + 10P\, v_{.07}^{10} + \cdots + 10P\, v_{.07}^{10}}_{= 10 \cdot 10P\, v_{.07}^{10} = 100P\, v_{.07}^{10}}$$

$$\therefore 50000 = P[(Ia)_{\overline{10}|.07} + 100 v_{.07}^{10}] = P\left[\frac{\ddot{a}_{\overline{10}|.07} - 10 v_{.07}^{10}}{.07} + 100 v_{.07}^{10}\right]$$
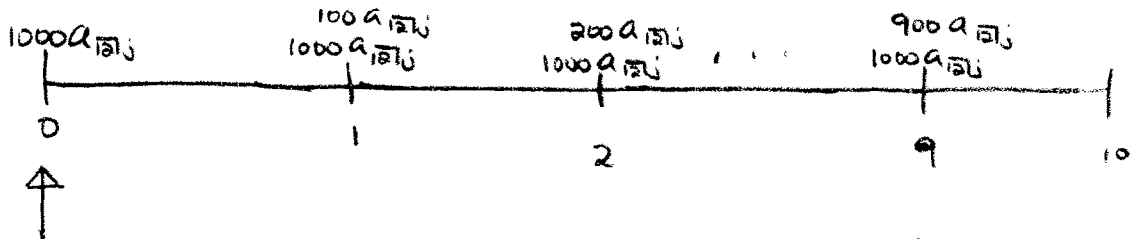
$$\Rightarrow P \doteq 584$$

25) Replace the lottery payments with the stream

$$j = \text{meir} \qquad (1+j)^{12} = 1.03 \Rightarrow j = .002466$$

The above payments are arithmetic with common
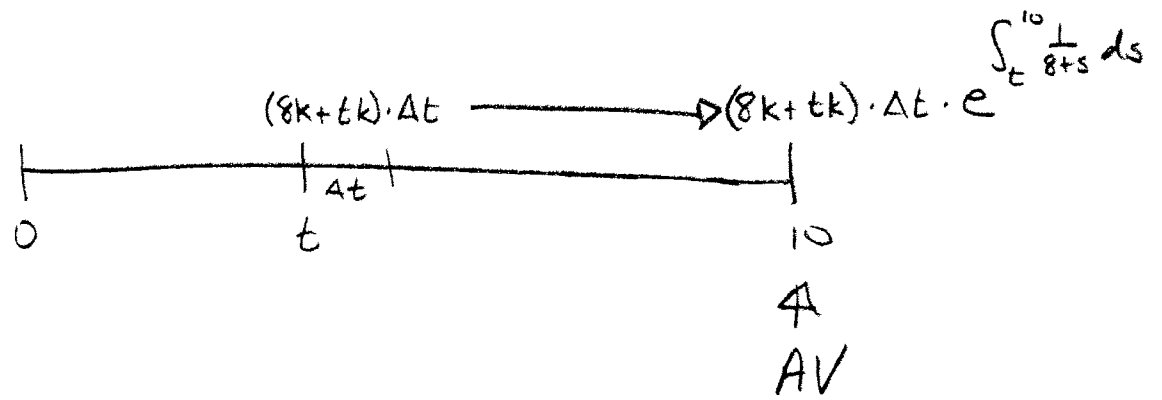difference $= 100\, a_{\overline{12}|j}$

Rewrite the stream as

$$\therefore PV = 1000\, a_{\overline{12}|j} \cdot \ddot{a}_{\overline{10}|.03} + 100\, a_{\overline{12}|j} \cdot (Ia)_{\overline{9}|.03}$$

$$= \frac{\ddot{a}_{\overline{9}|.03} - 9v_{.03}^{9}}{.03}$$

Calculations $\Rightarrow PV = 147929$

26) $\displaystyle\int_{0}^{n} \bar{a}_{\overline{t}|}\, dt = \int_{0}^{n} \frac{1 - v^{t}}{\delta}\, dt = \frac{1}{\delta} \int_{0}^{n} (1 - v^{t})\, dt$

$$= \frac{1}{\delta}\left[n - \int_{0}^{n} v^{t}\, dt \right] = \frac{1}{\delta}\left[n - \bar{a}_{\overline{n}|} \right] \xlongequal{\text{given}} 100$$

$$\Rightarrow \bar{a}_{\overline{n}|} = n - 100\delta$$

27)

$$(8k+tk)\cdot\Delta t \longrightarrow (8k+tk)\cdot\Delta t\cdot e^{\int_{t}^{10}\frac{1}{8+s}ds}$$

```
|----------|----------|----------|
0          t   Δt      10
```

$$\underset{AV}{\big\Uparrow}$$

$$AV = \int_{0}^{10}(8k+tk)\cdot e^{\int_{t}^{10}\frac{1}{8+s}ds}\,dt$$

$$= \int_{0}^{10}(8k+tk)\cdot e^{\ln(8+s)\big|_{t}^{10}}\,dt$$

$$= \int_{0}^{10}(8k+tk)\cdot e^{\ln\left(\frac{18}{8+t}\right)}\,dt$$

$$= \int_{0}^{10}k(8+t)\cdot\frac{18}{8+t}\,dt = 18k(10)=180K$$

$$AV = 20000$$

$$\Rightarrow 180k=20000 \Rightarrow k=111.11$$