Each problem is worth 10 points. Show sufficient work, use correct notation, and clearly mark your answer.

1. An increasing perpetuity immediate with annual payments has a present value of 860 at an annual effective discount rate, d. The initial payment at the end of the first year is 3 and each subsequent payment is 2 more than its preceding payment.

Determine d.

(A) .0476 (B) .0500 (C) .0526 (D) .0550 (E) .0576

2. The present value of the following three annuities are equal:

(i) Perpetuity-immediate paying 1 each year, calculated at an annual effective interest rate of 7.25%

(ii) 50-year annuity immediate paying 1 each year, calculated at an annual effective interest rate of j %.

(iii) n-year annuity immediate paying 1 each year, calculated at an annual effective interest rate of $(j-1)$ %.

Determine n.

(A) 30 (B) 33 (C) 36 (D) 39 (E) 42

3. Annuity A is a 10-year annuity with payments of 100 at the beginning of each quarter. Annuity B is a 10-year annuity with payments of 100 at the beginning of each quarter for the first year, 200 at the beginning of each quarter for the second year, and so on, with payments of 1000 at the beginning of each quarter for the tenth year. Using an annual effective interest rate of 6%, the present value of Annuity A is X and the present value of Annuity B is Y. Determine the value of $X + Y$.

(A) less than or equal to 18,200
(B) greater than 18,200 but less than or equal to 18,300
(C) greater than 18,300 but less than or equal to 18,400
(D) greater than 18,400 but less than or equal to 18,500
(E) greater than 18,500

4. A loan of 10,000 is being repaid with payments of 1000 at the end of each year for as long as necessary plus an additional smaller payment one year after the last payment of 1000. Using an effective interest rate of 6% per annum, determine the amount of the final payment.

(A) 610 (B) 650 (C) 690 (D) 730 (E) 770

5. A 20 year annuity-due with annual payments has a first payment of 100 and each subsequent payment is 7.1612% more than its preceding payment. Determine the present value of this annuity at a nominal interest rate of 4% compounded semiannually.

(A) 1430 (B) 1490 (C) 1530 (D) 2580 (E) 2690

6. An annuity immediate with annual payments has an initial payment of 1. Subsequent payments increase by 1 until reaching a payment of 10. The next payment after the payment of 10 is also equal to 10, and then subsequent payments decrease by 1 until reaching a final payment of 1. Determine the annual effective interest rate at which the present value of this annuity is 78.60.

(A) .0325 (B) .0335 (C) .0345 (D) .0355 (E) .0365

7. Phil began saving money for his retirement by making monthly deposits of 200 into a fund earning 6% interest compounded monthly. The first deposit occurred on January 1, 1985. Phil became unemployed and missed making deposits 60 through 72. He then continued making monthly deposits of 200.

Determine the amount accumulated in Phil's retirement fund on December 31, 1999?

(A) 53,572 (B) 53,590 (C) 53,840 (D) 53,860 (E) 54,205

8. A 10-year increasing annuity-immediate paying 5 in the first year and increasing by 5 each year thereafter has a present value of G. A 10-year decreasing annuity-immediate paying y in the first year and decreasing by $y/10$ each year thereafter has a present value of G. Both present values are calculated using an annual effective interest rate of 6%.

Determine y.

(A) 42 (B) 44 (C) 46 (D) 48 (E) 50

9. Deposits of 100 are made into an account at the beginning of each 4-year period. The account credits interest at an annual effective interest rate of i, $i > 0$. The accumulated amount in the account at the end of 40 years is 5 times the accumulated amount in the account at the end of 20 years. Using the same annual effective interest rate, a single payment of X has a present value 20 years before the payment of 40. Determine X.

(A) 120 (B) 160 (C) 200 (D) 240 (E) 280

10. Kay invest X at the beginning of each year for 10 years into an account that pays interest at the end of each year using an annual effective interest rate of 10%. She reinvests the interest payments each year into an account that pays an annual effective interest rate of 7%. At the end of 10 years, Kay has a total accumulated amount of principal plus interest equal to 1095. Determine X.

(A) 60 (B) 65 (C) 70 (D) 75 (E) 80

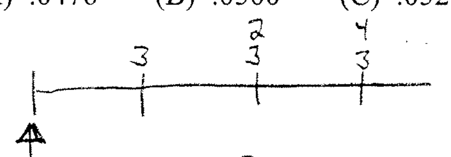
MAP 4170 Name: KEY
Test 2 Date: June 20, 2012

Each problem is worth 10 points. Show sufficient work, use correct notation, and clearly mark your answer.

1. An increasing perpetuity immediate with annual payments has a present value of 860 at an annual effective discount rate, d. The initial payment at the end of the first year is 3 and each subsequent payment is 2 more than its preceding payment.

Determine d.

(A) .0476 (B) .0500 (C) .0526 (D) .0550 (E) .0576



$i = aeir \quad d = aedr \quad d = \frac{i}{1+i}$

$$PV = \frac{3}{i} + \frac{2}{i^2} = 860 \Longrightarrow 860i^2 - 3i - 2 = 0$$

$$\Longrightarrow i = \frac{3 \pm \sqrt{9 - 4(860)(-2)}}{2(860)} = .05$$

$$\therefore d = \frac{.05}{1.05} \doteq .0476 \quad (A)$$

2. The present value of the following three annuities are equal:

(i) Perpetuity-immediate paying 1 each year, calculated at an annual effective interest rate of 7.25%

(ii) 50-year annuity immediate paying 1 each year, calculated at an annual effective interest rate of j %.

(iii) n-year annuity immediate paying 1 each year, calculated at an annual effective interest rate of $(j - 1)$ %.

Determine n.

(A) 30 (B) 33 (C) 36 (D) 39 (E) 42

$(i) \doteq (ii) \quad \frac{1}{.0725} = a_{\overline{50|}j} \Longrightarrow j \doteq .07$

$(i) \doteq (iii) \quad \frac{1}{.0725} = a_{\overline{n|}.06} \Longrightarrow n \doteq 30 \quad (A)$

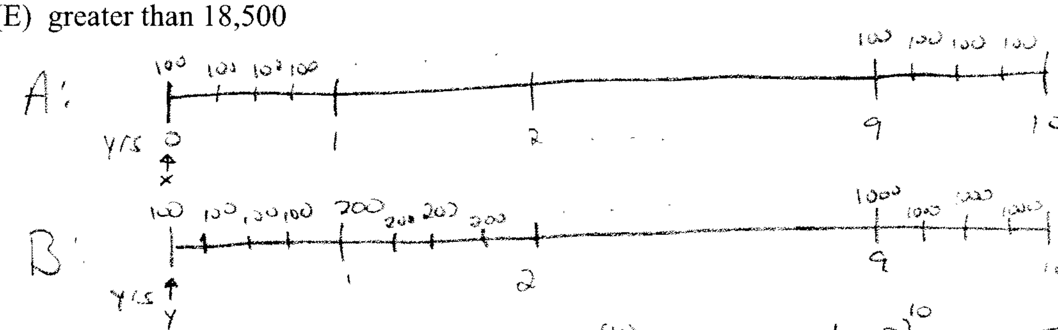
3. Annuity A is a 10-year annuity with payments of 100 at the beginning of each quarter. Annuity B is a 10-year annuity with payments of 100 at the beginning of each quarter for the first year, 200 at the beginning of each quarter for the second year, and so on, with payments of 1000 at the beginning of each quarter for the tenth year. Using an annual effective interest rate of 6%, the present value of Annuity A is X and the present value of Annuity B is Y. Determine the value of $X + Y$.

$i = .06 = aeir \Rightarrow j = qeir = 1.06^{1/4} - 1$

$d = qedr = \frac{j}{1+j}$

$d^{(4)} = 4d \doteq .057846553$

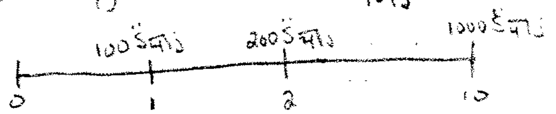
(A) less than or equal to 18,200
(B) greater than 18,200 but less than or equal to 18,300
(C) greater than 18,300 but less than or equal to 18,400
(D) greater than 18,400 but less than or equal to 18,500
(E) greater than 18,500



Using upper m notation: $X = 400\ddot{a}^{(4)}_{\overline{10|}.06} = 400\frac{1 - v^{10}_{.06}}{d^{(4)}} \doteq 3053.63$

$Y = 400(I\ddot{a})^{(4)}_{\overline{10|}.06} = 400\frac{\ddot{a}_{\overline{10|}.06} - 10v^{10}_{.06}}{d^{(4)}} \doteq 15335.36$

Alternatively, $X = 100\ddot{a}_{\overline{40|}j} = 3053.63$, and for Y, rewrite timeline as

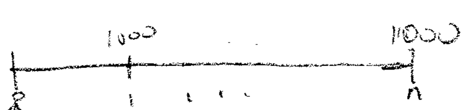


$Y = 100\ddot{s}_{\overline{4|}j}(Ia)_{\overline{10|}.06} = 100\ddot{s}_{\overline{4|}j}\frac{\ddot{a}_{\overline{10|}.06} - 10v^{10}_{.06}}{.06} \doteq 15335.36$

$\therefore X + Y \doteq 18389$ (C)

4. A loan of 10,000 is being repaid with payments of 1000 at the end of each year for as long as necessary plus an additional smaller payment one year after the last payment of 1000. Using an effective interest rate of 6% per annum, determine the amount of the final payment.

(A) 610 (B) 650 (C) 690 (D) 730 (E) 770



$10000 = 1000\, a_{\overline{n|}.06} \Rightarrow n = 15+$

The additional payment at the time of the last payment of 1000 can be computed as

$10000 = 1000\, a_{\overline{15|}.06} + X v^{15}_{.06} \Rightarrow X = 689.61$

$\therefore$ The addional payment one year later is $Y = 689.61(1.06) \doteq 730$ (D)

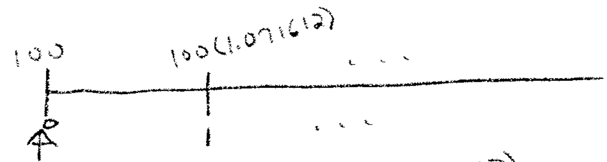
5. A 20 year annuity-due with annual payments has a first payment of 100 and each subsequent payment is 7.1612% more than its preceding payment. Determine the present value of this annuity at a nominal interest rate of 4% compounded semiannually.

(A) 1430 (B) 1490 (C) 1530 (D) 2580 (E) 2690

$i^{(2)} = .04$

$\Rightarrow .02 = \text{seir}$

$\Rightarrow j = 1.02^2 - 1 = .0404 = \text{ae}$



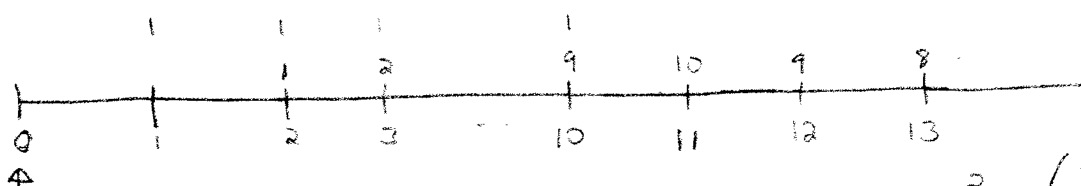
$$PV = 100 + 100\underbrace{\frac{(1.071612)}{1.0404}}_{=1.03} + \cdots \quad (20 \text{ terms})$$

$$= 100(1 + 1.03 + 1.03^2 + \cdots) = 100\, s_{\overline{20}|.03} = 2687.04$$

E

6. An annuity immediate with annual payments has an initial payment of 1. Subsequent payments increase by 1 until reaching a payment of 10. The next payment after the payment of 10 is also equal to 10, and then subsequent payments decrease by 1 until reaching a final payment of 1. Determine the annual effective interest rate at which the present value of this annuity is 78.60.

(A) .0325 (B) .0335 (C) .0345 (D) .0355 (E) .0365



$$PV = 78.60 = a_{\overline{10}|} + (\ddot{a}_{\overline{10}|})^2 \cdot v^2 = a_{\overline{10}|} + (a_{\overline{10}|})^2 \quad \left(\text{quadratic in } a_{\overline{10}|}\right)$$

$$\therefore\ a_{\overline{10}|} = \frac{-1 + \sqrt{1 - 4(1)(-78.6)}}{2(1)} = 8.379752249$$

$$\Rightarrow i \doteq .0335$$

B

7. Phil began saving money for his retirement by making monthly deposits of 200 into a fund earning 6% interest compounded monthly. The first deposit occurred on January 1, 1985. Phil became unemployed and missed making deposits 60 through 72. He then continued making monthly deposits of 200.

Determine the amount accumulated in Phil's retirement fund on December 31, 1999?

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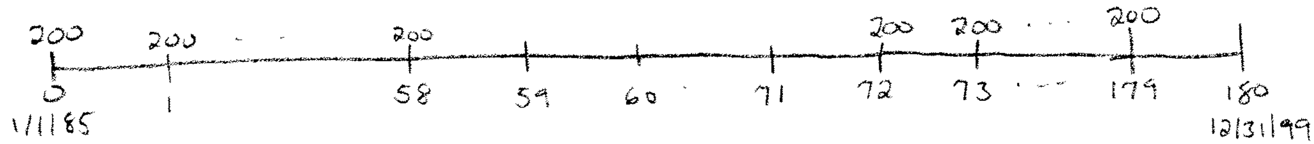


$i^{(12)} = .06 \Rightarrow i = .005 = \text{meir}$

$$AV = 200\,\ddot{s}_{\overline{180}|.005} - 200\,\ddot{s}_{\overline{13}|.005}(1.005)^{180-72} \doteq 53840$$

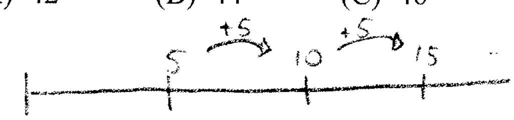
C

8. A 10-year increasing annuity-immediate paying 5 in the first year and increasing by 5 each year thereafter has a present value of G. A 10-year decreasing annuity-immediate paying y in the first year and decreasing by $y/10$ each year thereafter has a present value of G. Both present values are calculated using an annual effective interest rate of 6%.

Determine y.

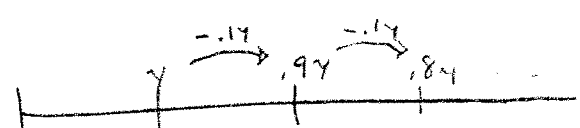
(A) 42 (B) 44 (C) 46 (D) 48 (E) 50

(Inc)



$$G = 5(Ia)_{\overline{10}|.06} = 5\,\frac{\ddot{a}_{\overline{10}|.06} - 10v^{10}_{.06}}{.06} \doteq 184.81$$

(Dec)



$$G \doteq 184.81 \doteq .1y\,(Da)_{\overline{10}|.06} = .1y\,\frac{10 - a_{\overline{10}|.06}}{.06}$$

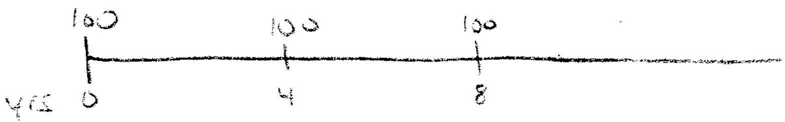
$$\Rightarrow y \doteq 42$$

A

9. Deposits of 100 are made into an account at the beginning of each 4-year period. The account credits interest at an annual effective interest rate of i, $i > 0$. The accumulated amount in the account at the end of 40 years is 5 times the accumulated amount in the account at the end of 20 years. Using the same annual effective interest rate, a single payment of X has a present value 20 years before the payment of 40. Determine X.

(A) 120 (B) 160 (C) 200 (D) 240 (E) 280

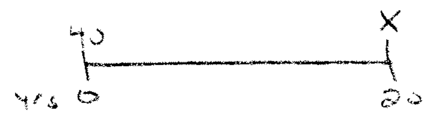
Let j = 4 year eir $\quad (1+i)^4 = 1+j$



$AV_{20} = 100\,\ddot{s}_{\overline{5}|j} \qquad AV_{40} = 100\,\ddot{s}_{\overline{10}|j} = 100\,\ddot{s}_{\overline{5}|j}\left(1+(1+j)^5\right)$

$\therefore\ 100\,\ddot{s}_{\overline{5}|j}\left(1+(1+j)^5\right) = 5\cdot 100\,\ddot{s}_{\overline{5}|j}$

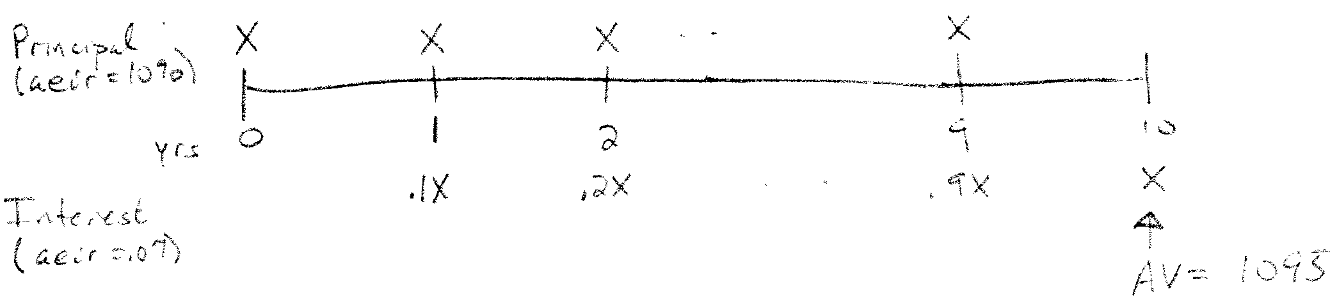
$\Longrightarrow (1+j)^5 = 4$



$X = 40(1+i)^{20} = 40(1+j)^5 = 40(4) = 160$ (B)

10. Kay invest X at the beginning of each year for 10 years into an account that pays interest at the end of each year using an annual effective interest rate of 10%. She reinvests the interest payments each year into an account that pays an annual effective interest rate of 7%. At the end of 10 years, Kay has a total accumulated amount of principal plus interest equal to 1095. Determine X.

(A) 60 (B) 65 (C) 70 (D) 75 (E) 80



$\therefore\ 1095 = 10X + .1X(Is)_{\overline{10}|.07} = 10X + .1X\,\dfrac{\ddot{s}_{\overline{10}|.07} - 10}{.07}$

$\Longrightarrow X \doteq 65$ (B)