Extra Problems for Test 1

1. You are given that $a(t) = Kt^2 + Lt + M$, for $0 \le t \le 2$, and that $a(0) = 100$, $a(1) = 110$, and $a(2) = 136$. Determine $\delta_{0.5}$.

a. .030 b. .049 c. .061 d. .095 e. .097

2. On July 1, 1999, a person invested 1000 in a fund for which the force of interest at time *t* is given by $\delta_t = \dfrac{3+2t}{50}$, where *t* is the number of years since January 1, 1999. Determine the accumulated value of the investment on January 1, 2000.

a. 1036 b. 1041 c. 1045 d. 1046 e. 1051

3. Investment X for 100,000 is invested at a nominal rate of interest, *j*, convertible semiannually. After four years it accumulates to 214,358.88. Investment Y for 100,000 is invested at a nominal rate of discount, *k*, convertible quarterly. After two years it accumulates to 232,305.73. Investment Z for 100,000 is invested at an annual effective interest rate equal to *j* in year one and an annual effective discount rate equal to *k* in year two. Calculate the value of investment Z at the end of two years.

a. 168,000 b. 182,900 c. 184,435 d. 200,000 e. 201,675

4. Fund X starts with 1000 and accumulates with force of interest $\delta_t = \dfrac{1}{15-t}$, for $0 < t < 15$.

Fund Y starts with 1000 and accumulates with an interest rate of 8% per annum compounded semiannually for the first three years and an effective interest rate of *i* per annum thereafter.

The amount in Fund X equals the amount in Fund Y at the end of four years. Calculate *i*.

a. .0750 b. .0775 c. .0800 d. .0825 e. .0850

Extra Problems for Test 1

5. At time $t = 0$, Billy puts 625 into an account paying 6% simple interest. At the end of year 2, George puts 400 into an account paying interest at a force of interest $\delta_t = \frac{1}{6+t}$, for $t \geq 2$. If both accounts continue to earn interest indefinitely at the levels given above, the amounts in the two accounts will be equal at the end of year n. Calculate n.

a. 23 b. 24 c. 25 d. 26 e. 27

6. You are given that $\delta_t = \frac{.2t}{1+.1t^2}$, for $t > 0$. Determine i_2.

a. .21 b. .24 c. .27 d. .31 e. .36

7. You are given a loan on which interest is charged over a 4-year period as follows:

(i) An effective rate of discount of 6% for the first year
(ii) A nominal rate of discount of 5% compounded every 2 years for the second year
(iii) A nominal rate of interest of 5% compounded semiannually for the third year
(iv) A force of interest of 5% for the fourth year

Calculate the annual effective rate of interest over the 4-year period.

a. .0500 b. .0525 c. .0550 d. .0575 e. .0600

8. Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns an annual effective discount rate of d.

The amount of interest earned in Bruce's account during the 11$^{\text{th}}$ year is equal to X. The amount of interest earned in Robbie's account during the 17$^{\text{th}}$ year is also equal to X.

Calculate X.

a. 28.0 b. 31.3 c. 34.6 d. 36.7 e. 38.9

9. Simplify the following expression: $\left(\frac{d}{dv}\delta\right)\left(\frac{d}{di}d\right)$.

a. $-v^3$ b. $-v$ c. 1 d. v e. v^3

Extra Problems for Test 1

10. Which of the following are true?

I. $\frac{d}{dd}(i) = v^{-2}$ II. $\frac{d}{di}(i^{(m)}) = v^{-\left(\frac{m-1}{m}\right)}$ III. $\frac{d}{d\delta}(i) = 1+i$

a. I and II only b. I and III only c. II and III only d. I, II, and III
e. The correct answer is not given by a., b., c., or d..

11. Payment of 300, 500, and 700 are made at the end of years five, six, and eight, respectively. Interest is accumulated at an annual effective interest rate of 4%. You are to find the point in time at which a single payment of 1500 is equivalent to the above series of payments. You are given:

(i) X is the point in time calculated using the method of equated time.
(ii) Y is the exact point in time.

Calculate $X + Y$.

a. 13.44 b. 13.50 c. 13.55 d. 14.61 e. 14.99

12. A loan of 1000 is made at an interest rate of 12% compounded quarterly. The loan is to be repaid with three payments: 400 at the end of the first year, 800 at the end of the fifth year, and the balance at the end of the tenth year. Calculate the amount of the final payment.

a. 587 b. 658 c. 737 d. 777 e. 812

13. The present value of a payment of 1004 at the end of T months is equal to the present value of 314 after 1 month, 271 after 18 months, and 419 after 24 months. The effective annual interest rate is 5%. Calculate T.

a. 14 b. 15 c. 16 d. 17 e. 18

Extra Problems for Test 1

14. An investor deposits 1000 on January 1 of year x and deposits 1000 on January 1 of year $x + 2$ into a fund that matures on January 1 of year $x + 4$. The interest rate on the fund differs every year and is equal to the annual effective rate of growth of the gross domestic product (GDP) during the fourth quarter of the previous year.

The following are the relevant GDP values for the past 4 years.

	Quarter	
Year	III	IV
$x - 1$	800.0	808.0
x	850.0	858.5
$x + 1$	900.0	918.0
$x + 2$	930.0	948.6

What is the internal rate of return earned by the investor over the 4-year period.

a. 1.66% b. 5.10% c. 6.15% d. 6.60% e. 6.78%

15. Project P requires an investment of 4000 at time 0.

The investment pays 2000 at time 1 and 4000 at time 2.

Project Q requires an investment of X at time 2. The investment pays 2000 at time 0 and 4000 at time 1. Using the net present value method and an interest rate of 10%, the net present values of the two projects are equal.

Calculate X

a. 5400 b. 5420 c. 5440 d. 5460 e. 5480

1) $a(0) = 100 \Rightarrow M = 100$

$a(1) = 110 \Rightarrow K + L + 100 = 110 \Rightarrow K + L = 10$ $\quad * -2$

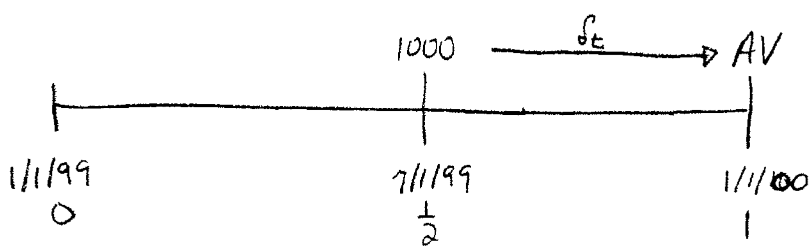
$a(2) = 136 \Rightarrow 4K + 2L + 100 = 136 \Rightarrow 4K + 2L = 36$

$$\begin{array}{rl} 4K+2L &= 36 \\ -2K-2L &= -20 \\ \hline 2K &= 16 \Rightarrow K = 8 \\ & \Rightarrow L = 2 \end{array}$$

$$\therefore \left.\begin{array}{l} a(t) = 8t^2 + 2t + 100 \\ a'(t) = 16t + 2 \end{array}\right\} \Rightarrow \delta_t = \frac{16t+2}{8t^2+2t+100}$$

$$\delta_{0.5} = \frac{16(0.5)+2}{8(0.5)^2+2(0.5)+100} = \frac{10}{103} \doteq .097$$

2)



$$AV = 1000\, e^{\int_{1/2}^{1} \frac{3+2t}{50}\,dt} = 1000 \cdot e^{\frac{3}{50}\left(\frac{1}{2}\right) + \left.\frac{t^2}{50}\right|_{1/2}^{1}}$$

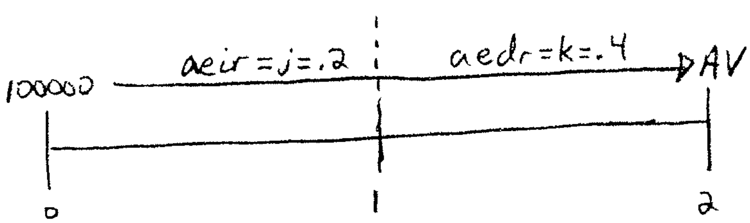
$$= 1000\, e^{\frac{3}{100} + \frac{1}{50} - \frac{1}{200}} \doteq 1046$$

3) $j = i^{(2)} \qquad k = d^{(4)}$

X: $214358.88 = 100000\left(1 + \frac{j}{2}\right)^8 \Rightarrow j \doteq 20\%$

Y: $232305.73 = 100000\left(1 - \frac{k}{4}\right)^{-8} \Rightarrow k \doteq 40\%$

Z:



$$AV = 100000\,(1.2)(.6)^{-1} = 200000$$

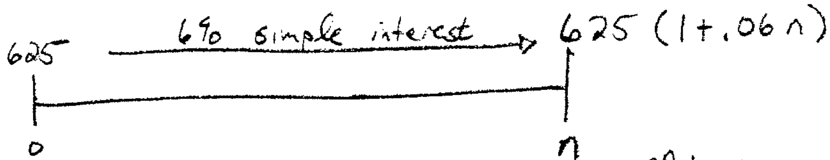
4) X: $AV_4 = 1000\, e^{\int_0^4 \frac{1}{15-t}\,dt} = 1000\, e^{-\ln(15-t)\big|_0^4}$

$$= 1000\, e^{\ln\left(\frac{15}{11}\right)} = \frac{15000}{11}$$

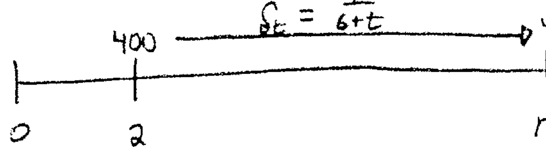
Y: $AV_4 = 1000(1.04)^6(1+i) = \frac{15000}{11}$

$$\Longrightarrow i = 7.77\%$$

5) B:



G:



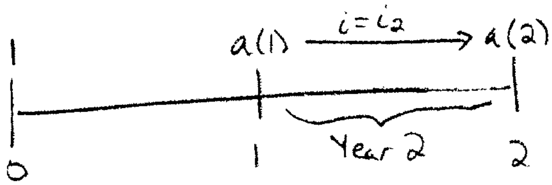
$400\, e^{\int_2^n \frac{1}{6+t}\,dt} = 400\, e^{\ln(6+t)\big|_2^n}$

$= 400\, \frac{6+n}{8}$

$$625(1+.06n) = 400\left(\frac{6+n}{8}\right)$$

$$625 + 37.5n = 300 + 50n \Longrightarrow n = 26$$

6) i_2 = aeir for year 2



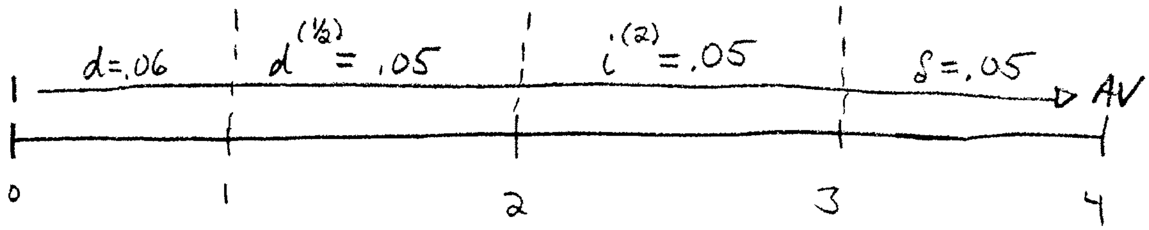
$a(1)\,(1+i) = a(2)$

$\Longrightarrow i = \frac{a(2)}{a(1)} - 1$

$$a(1) = e^{\int_0^1 \frac{.2t}{1+.1t^2}\,dt} = e^{\ln(1+.1t^2)\big|_0^1} = 1.1$$

$$a(2) = e^{\int_0^2 \frac{.2t}{1+.1t^2}\,dt} = e^{\ln(1+.1t^2)\big|_0^2} = 1.4$$

$$\therefore \quad i = \frac{1.4}{1.1} - 1 = \frac{.3}{1.1} \doteq .27$$

7) 

$i = $ aeir

$$(1+i)^4 = (1-.06)^{-1}\left(1-\frac{.05}{1/2}\right)^{-1/2}\left(1+\frac{.05}{2}\right)^2\left(e^{.05}\right) = 1.238549$$

$$\Longrightarrow i \doteq 5.49\%$$

8) Since the accounts earn interest at the same rate, from the information in the problem, we must have that the balance in Bruce's account at time 11 must equal the balance in Robbies account at time 17.

$$\therefore \quad 100(1-d)^{-11} = 50(1-d)^{-17}$$

$$\Longrightarrow \quad 2 = (1-d)^{-6} \Longrightarrow d \doteq .1091$$

Then $X = 100(1-d)^{-11} \cdot d$ (Recall that when using a periodic edr, we get the interest paid by multiplying the rate by the end of period balance)

$\doteq 38.9$

9) $\frac{d}{dv}\delta$: First write δ in terms of v

$$v = e^{-\delta} \Longrightarrow \delta = -\ln(v)$$

$$\therefore \frac{d}{dv}\delta = -\frac{1}{v}$$

$\frac{d}{di}d$: First write d in terms of i

$$d = iv = \frac{i}{1+i} \Longrightarrow \frac{d}{di}d = \frac{(1+i)(1) - i(1)}{(1+i)^2} = \frac{1}{(1+i)^2} = v^2$$

$$\therefore \left(\frac{d}{dv}\delta\right)\left(\frac{d}{di}d\right) = \frac{-1}{v}\cdot v^2 = -v$$

10) I. $\frac{d}{dd}(i)$; $i = \frac{d}{1-d} \Longrightarrow \frac{d}{dd}(i) = \frac{(1-d)(1) - d(-1)}{(1-d)^2} = \frac{1}{(1-d)^2} = \frac{1}{v^2} = v^{-2}$

II. $\frac{d}{di}(i^{(m)})$: $\left(1+\frac{i^{(m)}}{m}\right)^m = 1+i \Longrightarrow 1+\frac{i^{(m)}}{m} = (1+i)^{1/m}$

$$\Longrightarrow i^{(m)} = m\left[(1+i)^{1/m} - 1\right]$$

$$\therefore \frac{d}{di}(i^{(m)}) = m \cdot \frac{1}{m}(1+i)^{1/m - 1} = (1+i)^{\frac{1-m}{m}}$$

$$= (1+i)^{-\left(\frac{m-1}{m}\right)}$$

III. $\frac{d}{d\delta}(i)$: $1+i = e^{\delta} \Longrightarrow i = e^{\delta} - 1 \Longrightarrow \frac{d}{d\delta}(i) = e^{\delta} = 1+i$

I. & III. are true

II. is false

11) $X = \frac{300}{1500}(5) + \frac{500}{1500}(6) + \frac{700}{1500}(8) = 6.7\overline{3}$

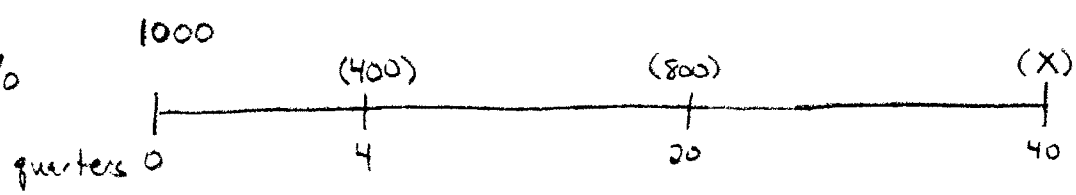
Y: $1500v^Y = 300v^5 + 500v^6 + 700v^8 \quad (v = (1.04)^{-1})$

$$1500v^Y = 1153.22$$

$$\Longrightarrow Y = \frac{\ln\left(\frac{1153.22}{1500}\right)}{-\ln(1.04)} \doteq 6.703$$

$$\therefore X + Y \doteq 13.44$$

12) $i^{(4)} = 12\%$



$$\left.\begin{array}{l} 1000 = 400v^4 + 800v^{20} + Xv^{40} \\ v = (1.03)^{-1} \end{array}\right\} \Longrightarrow X \doteq 658$$

13) $i = aeir = 5\%$

Find equivalent meir, j. $\Big\}$ $(1+j)^{12} = 1.05$

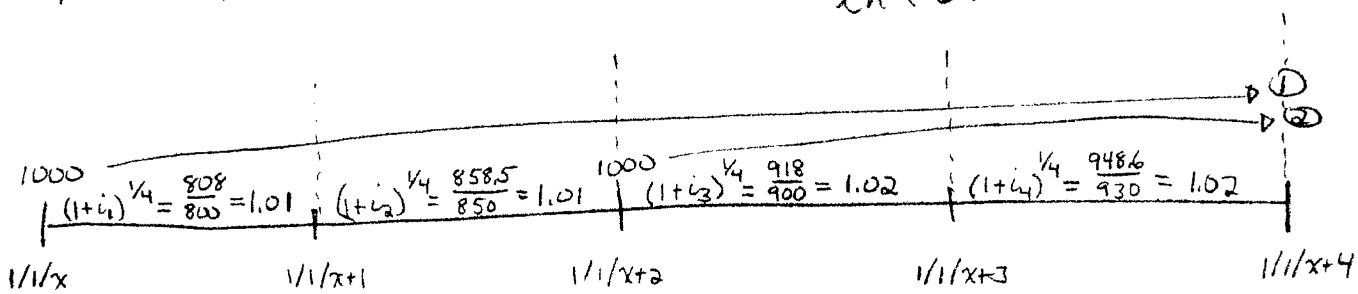
$\Rightarrow 1+j = (1.05)^{1/12}$

$$1004\,v^T = 314\,v + 271\,v^{18} + 419\,v^{24}$$

$$v = (1+j)^{-1} = (1.05)^{-1/12}$$

$$\therefore 1004\,v^T = 944.65 \Rightarrow T = \frac{\ln\left(\frac{944.65}{1004}\right)}{\ln(v)} \doteq 15$$

14)



$$① = 1000(1+i_1)(1+i_2)(1+i_3)(1+i_4) = 1000(1.01)^4(1.01)^4(1.02)^4(1.02)^4 = 1268.74$$

$$② = 1000\,(1+i_3)(1+i_4) = 1000\,(1.02)^4(1.02)^4 = 1171.66$$

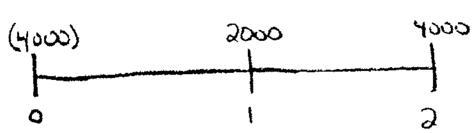
$$AV = ① + ② = 2440.4$$

$i = aeir$: $\quad 1000(1+i)^4 + 1000(1+i)^2 = 2440.4$ (quadratic in $(1+i)^2$)

$$1000(1+i)^4 + 1000(1+i)^2 - 2440.4 = 0$$

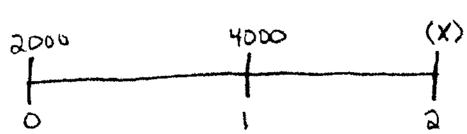
$$(1+i)^2 = \frac{-1000 \pm \sqrt{1000^2 - 4(1000)(2440.4)}}{2(1000)} \Rightarrow i \doteq 6.78\%$$

15) Project P:



$NPV_P = -4000 + 2000\,v + 4000\,v^2$

Project Q:



$NPV_Q = 2000 + 4000\,v - X\,v^2$

$$NPV_P = NPV_Q \Rightarrow -4000 + 2000\,v + 4000\,v^2 = 2000 + 4000\,v - X\,v^2$$

$$v = (1.1)^{-1}$$

$$\therefore X = 5460$$