Each problem is worth 10 points. Use correct notation and clearly mark your answers.

1. Account *A* earns interest using a 6% interest rate, compounded monthly. Account *B* earns interest according to $\delta_t = kt^2$. The forces of interest in the two accounts are equal at time $t = 2$. An amount X, deposited into Account *B* at time $t = 1$, has a value of 15 at time $t = 4$. Determine X.

   (A) Less than 11

   (B) Greater than or equal to 11, but less than 12

   (C) Greater than or equal to 12, but less than 13

   (D) Greater than or equal to 13, but less than 14

   (E) Greater than or equal to 14

2. Determine the nominal rate of interest compounded quarterly that is equivalent to a 6% discount rate, compounded semiannually.

   (A) 1.53%

   (B) 3.07%

   (C) 6.14%

   (D) 6.28%

   (E) 12.56%

3. For an account that credits interest using a simple interest rate, i, compute the ratio $\frac{i_{n+1}}{d_n}$, where i_{n+1} is the effective interest rate for year n+1, and d_n is the effective discount rate for year n.

   (A) 1

   (B) $\frac{i}{1+n}$

   (C) $\frac{i}{1+ni}$

   (D) $\frac{n}{1+ni}$

   (E) the correct answer is not given above

4. Amy takes out a 5-year loan from Bank A on 01/01/2010 based on a 5% simple discount rate starting on this date. Amy will pay back the loan in full at the end of 5 years. Although she doesn't recall the initial amount of the loan, on 01/01/2012 the balance on the loan is 4000. Determine the amount Amy will repay on 01/01/2015.

   (A) 4700

   (B) 4750

   (C) 4800

   (D) 4850

   (E) 4900

5. Determine the derivative, $\frac{d}{d\delta}(d)$, where d is the annual effective discount rate equivalent to the constant force of interest δ.

   (A) $-v^{-1}$

   (B) v^{-1}

   (C) $1 - v$

   (D) v

   (E) $-v$

6. The present value of a payment of $K paid at the end of 4 years is 150. The present value of $2K paid at the end of 8 years is also equal to 150. Determine the accumulated value of a payment of $K/2 over a 12 year period. All calculations use the same monthly effective discount rate.

   (A) 600

   (B) 900

   (C) 1200

   (D) 1500

   (E) 1800

7. An account credits interest using a 10% simple discount rate for the first two years of the investment and then using a 6% compounded semiannually interest rate. After being invested for 6 years, an amount *X* has accumulated to $2000. Determine *X*.

(A) 1200

(B) 1250

(C) 1300

(D) 1350

(E) 1400

8. An investment of $*K* is made in a fund for which $\delta_t = 1/(2t + 1)$ for $0 \leq t \leq 15$. The amount of interest earned on this investment from time $t = 4$ to $t = 12$ is 400. Determine *K*.

(A) 25

(B) 50

(C) 75

(D) 200

(E) 220

9. In return for payments of $5000 at the end of four years and $2000 at the end of ten years, an investor agrees to pay $3000 immediately and to make an additional payment at the end of three years. Find the amount of the additional payment if the nominal rate of discount converted quarterly is 6%.

   (A) 2340

   (B) 2360

   (C) 2380

   (D) 2400

   (E) 2420

10. At what nominal interest rate compounded monthly will payments of $100 at the end of one year, and $50 at the end of two years have a total present value of $112?

    (A) 1.9%

    (B) 3.8%

    (C) 16.9%

    (D) 22.5%

    (E) 25.0%

MAP 4170
Test 1

Name: KEY

Date: May 30, 2012

Each problem is worth 10 points. Use correct notation and clearly mark your answers.

1. Account A earns interest using a 6% interest rate, compounded monthly. Account B earns interest according to $\delta_t = kt^2$. The forces of interest in the two accounts are equal at time $t = 2$. An amount X, deposited into Account B at time $t = 1$, has a value of 15 at time $t = 4$. Determine X.

   (A) Less than 11

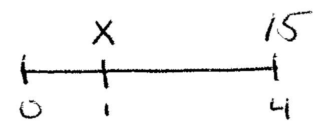
   (B) Greater than or equal to 11, but less than 12

   (C) Greater than or equal to 12, but less than 13

   (D) Greater than or equal to 13, but less than 14

   (E) Greater than or equal to 14

B:



$X = 15\, e^{\int_4^1 kt^2\,dt}$

A: $\delta_t = \delta = \ln(1+i)$ where i = a.e.r ; $1+i = (1.005)^{12}$

$\therefore \delta = 12\ln(1.005) = k(2)^2 \Rightarrow k = 3\ln(1.005)$

$\therefore X = 15\, e^{\frac{k}{3}t^3\big|_4^1} = 15\, e^{-21k} = 15\, e^{\ln(1.005)^{-63}}$

$= 15\,(1.005)^{-63} \doteq 10.96$

2. Determine the nominal rate of interest compounded quarterly that is equivalent to a 6% discount rate, compounded semiannually.

   (A) 1.53%

   (B) 3.07%

   (C) 6.14%

   (D) 6.28%

   (E) 12.56%

Accumulating 1 over one semiannual period gives

$\left(1 + \frac{i^{(4)}}{4}\right)^2 = \left(1 - \frac{.06}{2}\right)^{-1}$

$\Rightarrow i^{(4)} \doteq 6.14\%$

3. For an account that credits interest using a simple interest rate, i, compute the ratio $\frac{i_{n+1}}{d_n}$, where i_{n+1} is the effective interest rate for year n+1, and d_n is the effective discount rate for year n.

$a(t) = 1 + it$

$$i_{n+1} = \frac{a(n+1) - a(n)}{a(n)}$$

$$d_n = \frac{a(n) - a(n-1)}{a(n)}$$

$$a(n+1) - a(n) = i = a(n) - a(n-1)$$

$$\therefore \frac{i_{n+1}}{d_n} = 1$$

(A) 1

(B) $\frac{i}{1+n}$

(C) $\frac{i}{1+ni}$

(D) $\frac{n}{1+ni}$

(E) the correct answer is not given above

4. Amy takes out a 5-year loan from Bank A on 01/01/2010 based on a 5% simple discount rate starting on this date. Amy will pay back the loan in full at the end of 5 years. Although she doesn't recall the initial amount of the loan, on 01/01/2012 the balance on the loan is 4000. Determine the amount Amy will repay on 01/01/2015.

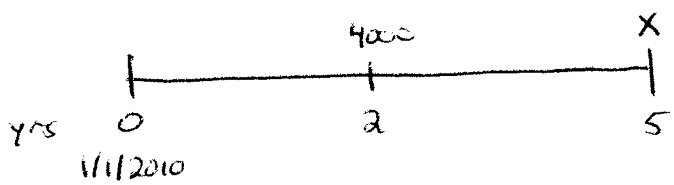
(A) 4700

(B) 4750

(C) 4800

(D) 4850

(E) 4900



$a(t) = (1 - .05t)^{-1}$

$$X = 4000 \cdot \frac{a(5)}{a(2)} = 4000 \frac{.75^{-1}}{.9^{-1}} = 4800$$

5. Determine the derivative, $\frac{d}{d\delta}(d)$, where d is the annual effective discount rate equivalent to the constant force of interest δ.

(A) $-v^{-1}$

(B) v^{-1}

(C) $1-v$

(D) v

(E) $-v$

$$d = 1 - e^{-\delta}$$

$$\Rightarrow \frac{d}{d\delta}(d) = \frac{d}{d\delta}\left[1 - e^{-\delta}\right] = e^{-\delta} = v$$

6. The present value of a payment of \$K paid at the end of 4 years is 150. The present value of \$2K paid at the end of 8 years is also equal to 150. Determine the accumulated value of a payment of \$K/2 over a 12 year period. All calculations use the same monthly effective discount rate.

(A) 600

(B) 900

(C) 1200

(D) 1500

(E) 1800

Let v = annual discount factor (could have used monthly)

$$150 = Kv^4 = 2Kv^8$$

$$\Rightarrow v^4 = \frac{1}{2} \quad \text{and} \quad K = 300$$

$$AV = \frac{K}{2} v^{-12} = \frac{K}{2}\left(v^4\right)^{-3} = \frac{K}{2}\left(\frac{1}{2}\right)^{-3}$$

$$= \frac{K}{2}(8) = 4K$$

$$= 1200$$

7. An account credits interest using a 10% simple discount rate for the first two years of the investment and then using a 6% compounded semiannually interest rate. After being invested for 6 years, an amount X has accumulated to \$2000. Determine X.

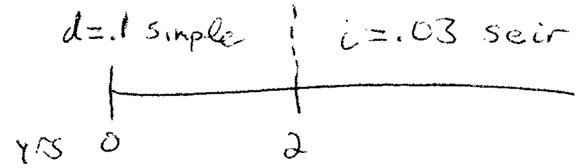
(A) 1200

(B) 1250

(C) 1300

(D) 1350

(E) 1400



$$X(1-.1(2))^{-1}(1.03)^8 = 2000$$

$$\Rightarrow X = 1263.05$$

8. An investment of \$$K$ is made in a fund for which $\delta_t = 1/(2t+1)$ for $0 \le t \le 15$. The amount of interest earned on this investment from time t = 4 to t = 12 is 400. Determine K.

(A) 25

(B) 50

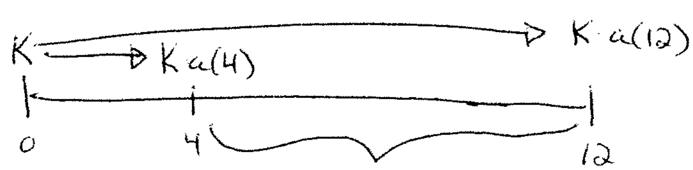
(C) 75

(D) 200

(E) 220

$$\delta_t = \frac{1}{2}\cdot\frac{2}{2t+1} \xrightarrow[\substack{f(t)=2t+1 \\ c=\frac{1}{2}}]{\text{Special Case}} a(t) = (2t+1)^{1/2}$$

$$a(t) = \sqrt{2t+1}$$



Amount of interest earned from $t=4$ to $t=12$ equals

$$K\cdot a(12) - K\cdot a(4) = 400$$

$$a(12) = \sqrt{25} = 5$$

$$a(4) = \sqrt{9} = 3$$

$$\Rightarrow 5K - 3K = 400$$

$$\Rightarrow K = 200$$

9. In return for payments of $5000 at the end of four years and $2000 at the end of ten years, an investor agrees to pay $3000 immediately and to make an additional payment at the end of three years. Find the amount of the additional payment if the nominal rate of discount converted quarterly is 6%.

(A) 2340

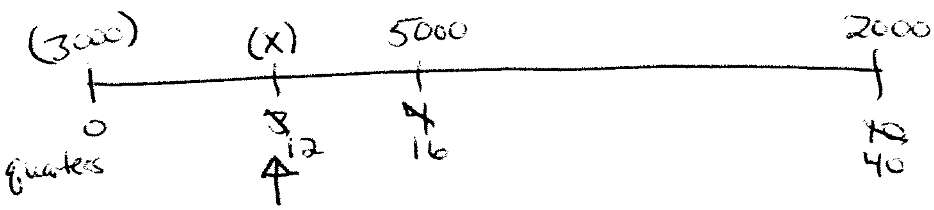
(B) 2360

(C) 2380

(D) 2400

(E) 2420

$d = \frac{.06}{4} = .015 = \text{qedr}$



$$X + 3000(1-d)^{-12} = 5000(1-d)^{4} + 2000(1-d)^{28}$$

$1-d = .985$

$\therefore X \doteq 2420$

10. At what nominal interest rate compounded monthly will payments of $100 at the end of one year, and $50 at the end of two years have a total present value of $112?

(A) 1.9%

(B) 3.8%

(C) 16.9%

(D) 22.5%

(E) 25.0%

Let v = annual discount factor

$$112 = 100v + 50v^2$$

$$\Rightarrow 50v^2 + 100v - 112 = 0$$

$$v = \frac{-100 \pm \sqrt{32400}}{2(50)} = 0.8$$

$$\Rightarrow 1+i = \frac{1}{0.8} = 1.25 \Rightarrow i = .25 = \text{aeir}$$

$$\therefore \left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1.25 \Rightarrow i^{(12)} \doteq 22.5\%$$