MAP 4170 – 01 Name:________________________________
Test 1 Date: May 25, 2011

Each problem is worth 10 points. Clearly mark your answers.

1. At an interest rate of i compounded monthly, the sum of the present values of 200 at the end of n years and 800 at the end of $2n$ years is 300. Determine the accumulated value of 100 after $3n$ years using the same interest rate.

   (A) 600

   (B) 650

   (C) 700

   (D) 750

   (E) 800

2. Deposits of 4000 at time $t = 2$, 2000 at time $t = 6$, and 4000 at time $t = 12$ are equivalent to a single deposit of 10000 at time X under the method of equated time. Determine X.

   (A) 6.2

   (B) 6.4

   (C) 6.6

   (D) 6.8

   (E) 7.0

3. In return for payments of $6000 on January 1, 2016, and $3000 on January 1, 2022, an investor agrees to pay $5000 on January 1, 2012, and to make an additional payment on January 1, 2015. The additional payment is to be calculated using an interest rate of 5%. Smith and Jones have both been charged with finding the amount of the additional payment. Smith assumes the given interest rate is simple, but Jones assumes it is compounded annually. Both use January 1, 2012, as the valuation date (this date corresponds to $t = 0$). Smith determines the additional payment to be X, and Jones determines the additional payment to be Y. Determine $X - Y$.

   (A) -280

   (B) -150

   (C) -20

   (D) 110

   (E) 240

4. Determine $\frac{dv}{d\delta}$

   (A) $-\delta$

   (B) δ

   (C) $-v$

   (D) v

   (E) None of the above

5. You are given:

   1) $\delta_t = \frac{t}{t^2+k}$ for some constant k and $t \geq 0$.
   2) 100 invested at time t = 0 accumulates to 125 at time t = 3.
   3) The discounted value at time t = 3 of a payment of 447 at time t = 8 is X.

   Determine X.

   (A) 160

   (B) 180

   (C) 200

   (D) 230

   (E) 250

6. Account A credits interest using an 8% simple interest rate. Account B credits interest using an 8% interest rate, convertible annually. Determine the time t (in years) at which the forces of interest in the accounts are equal.

   (A) 0.494

   (B) 0.544

   (C) 0.594

   (D) 0.644

   (E) 0.694

7. Account A credits interest using 6%, compounded monthly. Account B credits interest using a nominal rate of discount d, compounded quarterly. 100 is invested in Account A at the same time that 200 is invested in Account B. After 5 years the amount in Account A is again 100 less than the amount in Account B. Determine d.

   (A) 0.8%

   (B) 1.6%

   (C) 2.4%

   (D) 3.2%

   (E) 4.0%

8. You are given an account in which interest is credited over a 2-year period as follows:

   A nominal rate of discount of 5% compounded every 2 years is used for year 1.
   A force of interest of 5% is used for year 2.

   A deposit of 100 is made into the account at the beginning of each year for the 2 year period. The amount of interest earned in year 1 is I_1 and the amount of interest earned in year 2 is I_2. Determine the ratio $\frac{I_2}{I_1}$.

   (A) 1.79

   (B) 1.87

   (C) 1.95

   (D) 2.03

   (E) 2.11

9. An account credits interest using a simple discount rate of 5% over a 15-year period. Determine the annual effective interest rate during the tenth year for this account.

   (A) 9%

   (B) 10%

   (C) 11%

   (D) 12%

   (E) 13%

10. Determine the nominal rate of interest compounded quarterly that is equivalent to a nominal rate of discount of 18.9% compounded monthly.

    (A) 18.6%

    (B) 18.9%

    (C) 19.2%

    (D) 19.5%

    (E) 19.8%

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Test 1 Date: May 25, 2011

Each problem is worth 10 points. Clearly mark your answers.

1. At an interest rate of i compounded monthly, the sum of the present values of 200 at the end of n years and 800 at the end of $2n$ years is 300. Determine the accumulated value of 100 after $3n$ years using the same interest rate.

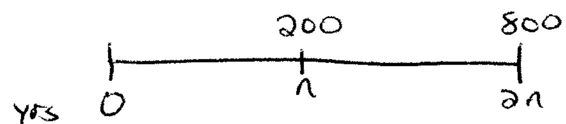
   (A) 600

   (B) 650

   (C) 700

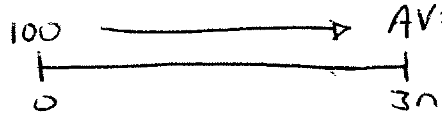
   (D) 750

   (E) 800



$PV = 200 v_j^n + 800 v_j^{2n} = 300$ (quadratic in v^n)

$\Rightarrow v^n = .5 \Rightarrow (1+j)^n = 2$ j = aeir



$AV = 100(1+j)^{3n} = 100 \cdot 2^3 = 800$

2. Deposits of 4000 at time $t = 2$, 2000 at time $t = 6$, and 4000 at time $t = 12$ are equivalent to a single deposit of 10000 at time X under the method of equated time. Determine X.

   (A) 6.2

   (B) 6.4

   (C) 6.6

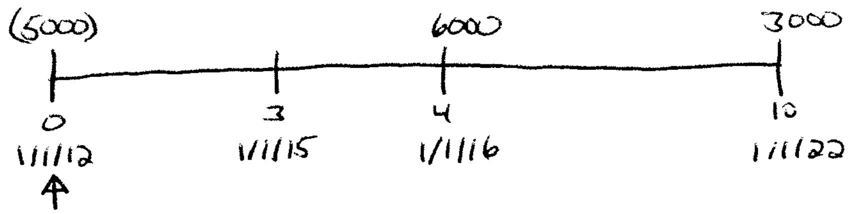
   (D) 6.8

   (E) 7.0

$$X = \frac{4000}{10000}(2) + \frac{2000}{10000}(6) + \frac{4000}{10000}(12)$$

$$= 6.8$$

3. In return for payments of \$6000 on January 1, 2016, and \$3000 on January 1, 2022, an investor agrees to pay \$5000 on January 1, 2012, and to make an additional payment on January 1, 2015. The additional payment is to be calculated using an interest rate of 5%. Smith and Jones have both been charged with finding the amount of the additional payment. Smith assumes the given interest rate is simple, but Jones assumes it is compounded annually. Both use January 1, 2012, as the valuation date (this date corresponds to $t = 0$). Smith determines the additional payment to be X, and Jones determines the additional payment to be Y. Determine $X - Y$.



(A) -280

(B) -150

(C) -20

(D) 110

(E) 240

$S: \ a(t) = 1 + .05t$

$$5000 + \frac{X}{a(3)} = \frac{6000}{a(4)} + \frac{3000}{a(10)} \Rightarrow X = 2300$$

$J: \ a(t) = 1.05^t \Rightarrow v = \frac{1}{1.05}$

$$5000 + Y v^3 = 6000 v^4 + 3000 v^{10} \Rightarrow Y \doteq 2060$$

$$X - Y \doteq 240$$

4. Determine $\frac{dv}{d\delta}$

(A) $-\delta$

(B) δ

(C) $-v$

(D) v

(E) None of the above

$$v = e^{-\delta}$$

$$\Rightarrow v' = -e^{-\delta} = -v$$

5. You are given:

   1) $\delta_t = \frac{t}{t^2+k}$ for some constant k and $t \geq 0$.
   2) 100 invested at time $t = 0$ accumulates to 125 at time $t = 3$.
   3) The discounted value at time $t = 3$ of a payment of 447 at time $t = 8$ is X.

   Determine X.

   $$\delta_t = \frac{1}{2} \cdot \frac{2t}{t^2+k} \Rightarrow a(t) = \sqrt{\frac{t^2+k}{k}}$$

   (A) 160

   (B) 180

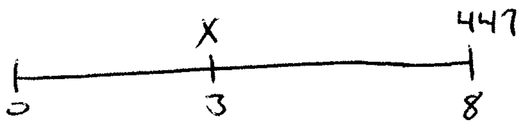
   $$125 = 100 \cdot a(3) = 100\sqrt{\frac{9+k}{k}} \Rightarrow k = 16$$

   (C) 200

   $$\therefore \; a(t) = \sqrt{\frac{t^2+16}{16}}$$

   (D) 230

   (E) 250



$$X = 447 \cdot \frac{a(3)}{a(8)} = 447 \cdot \frac{1.25}{\sqrt{5}} \doteq 250$$

6. Account A credits interest using an 8% simple interest rate. Account B credits interest using an 8% interest rate, convertible annually. Determine the time t (in years) at which the forces of interest in the accounts are equal.

   (A) 0.494

   (B) 0.544

   $$A: \quad a(t) = 1 + .08t \Rightarrow \delta_t = \frac{a'(t)}{a(t)} = \frac{.08}{1+.08t}$$

   (C) 0.594

   (D) 0.644

   $$B: \; a(t) = 1.08^t \Rightarrow \delta_t = \ln(1.08)$$

   (E) 0.694

   t - years

$$\frac{.08}{1+.08t} = \ln(1.08) \Rightarrow t \doteq 0.494$$

7. Account A credits interest using 6%, compounded monthly. Account B credits interest using a nominal rate of discount d, compounded quarterly. 100 is invested in Account A at the same time that 200 is invested in Account B. After 5 years the amount in Account A is again 100 less than the amount in Account B. Determine d.

(A) 0.8%

(B) 1.6%

(C) 2.4%

(D) 3.2%

(E) 4.0%

A: $100 \xrightarrow{i^{(12)} = .06} 100(1.005)^{60}$

yrs 0 — 5 (60 months)

B: $200 \xrightarrow{d = d^{(4)}} 200\left(1-\frac{d}{4}\right)^{-20}$

yrs 0 — 5 (20 quarters)

$$\therefore \quad 100(1.005)^{60} + 100 = 200\left(1-\frac{d}{4}\right)^{-20}$$

$$\Longrightarrow d \doteq 0.032$$

8. You are given an account in which interest is credited over a 2-year period as follows:

A nominal rate of discount of 5% compounded every 2 years is used for year 1.
A force of interest of 5% is used for year 2.

A deposit of 100 is made into the account at the beginning of each year for the 2 year period. The amount of interest earned in year 1 is I_1 and the amount of interest earned in year 2 is I_2. Determine the ratio $\frac{I_2}{I_1}$.

(A) 1.79

(B) 1.87

(C) 1.95

(D) 2.03

(E) 2.11

$100 \xrightarrow{d^{(1/2)} = .05} 100(1-.1)^{-1/2} \doteq 105.41$

0 — 1

$$\therefore I_1 = 5.41$$

For year 2, $105.41 + 100 = 205.41$ is invested.

$205.41 \xrightarrow{\delta = .05} 205.41e^{.05} \doteq 215.94$

1 — 2

$$\therefore I_2 \doteq 215.94 - 205.41 = 10.53$$

$$\therefore \frac{I_2}{I_1} \doteq 1.95$$

9. An account credits interest using a simple discount rate of 5% over a 15-year period. Determine the annual effective interest rate during the tenth year for this account.

(A) 9%

(B) 10%

(C) 11%

(D) 12%

(E) 13%

$$a(t) = (1 - .05t)^{-1}$$

$$i_{10} = \frac{a(10) - a(9)}{a(9)}$$

$$a(9) = .55^{-1}$$

$$a(10) = .5^{-1} = 2$$

$$\therefore \quad i_{10} = \frac{2 - .55^{-1}}{.55^{-1}} = 10\%$$

10. Determine the nominal rate of interest compounded quarterly that is equivalent to a nominal rate of discount of 18.9% ompounded monthly.

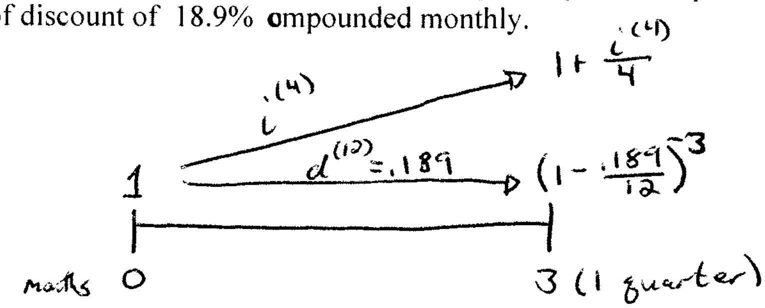
(A) 18.6%

(B) 18.9%

(C) 19.2%

(D) 19.5%

(E) 19.8%



$$1 + \frac{i^{(4)}}{4} = \left(1 - \frac{.189}{12}\right)^{-3}$$

$$\Rightarrow i^{(4)} \doteq 19.5\%$$