Section 3 Examples

1. You are given a perpetual annuity-immediate with annual payments increasing in geometric progression, with a common ratio of 1.07. The annual effective interest rate is 12%. The first payment is 1. Calculate the present value of this annuity.

a. 18 b. 19 c. 20 d. 21 e. 22

2. An annuity-immediate pays an initial benefit of 1 per year, increasing by 10.25% every four years. The annuity is payable for 40 years. Using an annual effective interest rate of 5%, determine an expression for the present value of this annuity.

a. $\left(1+\upsilon^2\right)\cdot \ddot{a}_{\overline{20|}}$ b. $\left(1+\upsilon^2\right)\cdot a_{\overline{20|}}$ c. $2\cdot a_{\overline{20|}}$ d. $\dfrac{a_{\overline{20|}}}{s_{\overline{2|}}}$ e. $\dfrac{a_{\overline{40|}}}{a_{\overline{2|}}}$

3. Chris makes annual deposits into a bank account at the beginning of each year for 20 years. Chris' initial deposit is equal to 100, with each subsequent deposit k% greater than the previous year's deposit. The bank credits interest at an annual effective interest rate of 5%.

At the end of 20 years, the accumulated value in Chris' account is equal to 7276.35.

Given $k > 5$, calculate k.

a. 8.06 b. 8.21 c. 8.36 d. 8.51 e. 8.68

4. Barbara purchases an increasing perpetuity with payments occurring at the end of every 2 years. The first payment is 1, the second one is 2, the third one is 3, and so on. The price of the perpetuity is 110. Calculate the annual effective interest rate.

a. 4.50% b. 4.62% c. 4.75% d. 4.88% e. 5.00%

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5. Joe can purchase one of two annuities:

Annuity 1: A 10-year decreasing annuity-immediate, with annual payments 10, 9, …, 1.

Annuity 2: A perpetuity-immediate with annual payments. The perpetuity pays 1 in year 1, 2 in year 2, …, and 11 in year 11. After year 11, the payments remain constant at 11.

At an annual effective interest rate of i, the present value of Annuity 2 is twice the present value of Annuity 1.

Calculate the present value of Annuity 1.

a. 36.4 b. 37.4 c. 38.4 d. 39.4 e. 40.4

6. Olga buys a 5-year increasing annuity for X.

Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2.

The nominal interest rate is 9% convertible quarterly.

Calculate X.

a. 2680 b. 2730 c. 2780 d. 2830 e. 2880

7. A varying immediate annuity with a term of $2n$ years has a first payment equal to 1. Thereafter payments increase by 1 each year until they reach n at the end of n years. Payments remain at n for year n+1, and then decrease by 1 each year, with a final payment of 1 at the end of $2n$ years. Derive an expression for the present value of this annuity.

a. $a_{\overline{n}|}\left(\frac{1}{d}-\frac{v^n}{i}\right)$ b. $a_{\overline{n}|}\left(\frac{1}{d}+\frac{v^n}{i}\right)$ c. $a_{\overline{n}|}\left(\frac{1}{d}-\frac{v^n}{d}\right)$

d. $a_{\overline{n}|}\left(\frac{1}{d}+\frac{v^n}{d}\right)$ e. $a_{\overline{n}|}\left(\frac{1}{i}+\frac{v^n}{d}\right)$

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8. An 11-year annuity has a series of payments 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, with the first payment made at the end of the second year. The present value of this annuity is 25 at interest rate i.

A 12-year annuity has a series of payments 1, 2, 3, 4, 5, 6, 6, 5, 4, 3, 2, 1, with the first payment made at the end of the first year.

Calculate the present value of the 12-year annuity at interest rate i.

a. 29.5 b. 30.0 c. 30.5 d. 31.0 e. 31.5

9. Mary purchases an increasing annuity-immediate for 50,000 that makes twenty annual payments as follows:

(i) P, $2P$, …, $10P$ in years 1 through 10; and

(ii) $10(1.05)P$, $10(1.05^2)P$, …, $10(1.05^{10})P$ in years 11 through 20.

The annual effective interest rate is 7% for the first 10 years, and 5% thereafter.

Calculate P.

a. 564 b. 574 c. 584 d. 594 e. 604

10. Joan has won a lottery that pays 1000 per month in the first year, 1100 per month in the second year, 1200 per month in the third year, and so on. Payments are made at the end of each month for 10 years.

Using an effective interest rate of 3% per annum, calculate the present value of this prize.

a. 107,000 b. 114,000 c. 123,000 d. 135,000 e. 148,000

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11. You are given $\int_0^n \bar{a}_{\overline{t}|} dt = 100$. Calculate $\bar{a}_{\overline{n}|}$.

a. $100n\delta$ b. $n\delta$ c. $n - 100\delta$ d. $100 - n\delta$ e. $n - \frac{\delta}{100}$

12. Payments are made to an account at a continuous rate of $(8k + tk)$, where $0 \le t \le 10$.

Interest is credited at a force of interest of $\delta_t = \frac{1}{8+t}$.

After 10 years, the account is worth 20,000.

Calculate *k*.

a. 111 b. 116 c. 121 d. 126 e. 131

Section 3 Key

1. C
2. B
3. C
4. D
5. D
6. B
7. A
8. B
9. C
10. E
11. C
12. A