

Section 2 Examples

1. A person deposits 100 at the beginning of each year for 20 years. Simple interest at an annual rate of i is credited to each deposit from the date of deposit to the end of the twenty year period. The total amount thus accumulated is 2840. If instead, compound interest had been credited at an effective annual rate of i , what would the accumulated value of these deposits have been at the end of twenty years?

- a. 2980 b. 3100 c. 3200 d. 3310 e. 3470

2. A perpetuity pays 1 at the end of every year plus an additional 1 at the end of every second year. The present value of the perpetuity is K for $i \geq 0$. Determine K .

- a. $\frac{i+3}{i(i+2)}$ b. $\frac{i+2}{i(i+1)}$ c. $\frac{i+1}{i^2}$ d. $\frac{3}{2i}$ e. $\frac{i+1}{i(i+2)}$

3. An annuity pays 1 at the end of each 4-year period for 40 years. Given $a_{\overline{8}|i} = k$, find the present value of the annuity.

- a. $\frac{1-(1-ik)^5}{1-(1-ik)^5}$ b. $\frac{1-(1-ik)^{40}}{1-(1-ik)^5}$ c. $\frac{1-(1-ik)^5}{i}$
 d. $\frac{1-(1-ik)^{40}}{(1-ik)^{-5} - 1}$ e. $\frac{1-(1-ik)^5}{(1-ik)^{-5} - 1}$

4. You are given $\delta_t = \frac{2}{10+t}$, $t \geq 0$. Calculate $a_{\overline{4}|}$.

- a. 2.34 b. 2.62 c. 2.85 d. 3.01 e. 3.23

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5. You are given two series of payments. Series A is a perpetuity with payments of 1 at the end of each of the first 2 years, 2 at the end of each of the next 2 years, 3 at the end of each of the next 2 years, and so on. Series B is a perpetuity with payments of K at the end of each of the first 3 years, $2K$ at the end of the next 3 years, $3K$ at the end of each of the next 3 years, and so on. The present values of the two series of payments are equal. Calculate K .

a. $\frac{3i}{2}$

b. $\frac{3d}{2}$

c. $\frac{a_{\overline{3}|}}{a_{\overline{2}|}}$

d. $\frac{a_{\overline{3}|}}{\ddot{a}_{\overline{2}|}}$

e. $\frac{\ddot{s}_{\overline{3}|}}{\ddot{s}_{\overline{2}|}}$

6. You are given an annuity-immediate with 11 annual payments of 100 and a final larger payment at the end of 12 years. At an annual effective interest rate of 3.5%, the present value at time 0 of all payments is 1000.

Using an annual effective interest rate of 1%, calculate the present value at the beginning of the ninth year of all remaining payments.

a. 412

b. 419

c. 432

d. 439

e. 446

7. The proceeds of a 10,000 death benefit are left on deposit with an insurance company for seven years at an effective annual interest rate of 5%.

The balance at the end of seven years is paid to the beneficiary in 120 equal monthly payments of X , with the first payment made immediately. During the payout period, interest is credited at an annual effective rate of 3%.

Calculate X .

a. 117

b. 118

c. 129

d. 135

e. 158

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8. You are given:

(i) The present value of an annuity-due that pays 300 every 6 months during the first 15 years and 200 every 6 months during the second 15 years is 6000.

(ii) The present value of a 15-year deferred annuity-due that pays 350 every 6 months for 15 years is 1580.

(iii) The present value of an annuity-due that pays 100 every 6 months during the first 15 years and 200 every 6 months during the next 15 years is X .

The same interest rate is used in all calculations. Determine X .

- a. 2302 b. 2402 c. 2502 d. 2602 e. 2702

9. At a nominal rate of interest i , convertible semiannually, the present value of a series of payments of 1 at the end of every 2 years forever, is 5.89. Calculate i .

- a. 6% b. 7% c. 8% d. 9% e. 10%

10. John deposits 100 at the end of each year for 20 years into a fund earning an annual effective interest rate of 7%.

Mary makes 20 deposits into a fund at the end of each year for 20 years. The first 10 deposits are 100 each, while the last 10 deposits are $100 + X$ each. The fund earns an annual effective interest rate of 8% during the first 10 years and 6% annual effective interest thereafter.

At the end of 20 years, the amount in John's fund equals the amount in Mary's fund.

Calculate X .

- a. 8 b. 10 c. 12 d. 14 e. 16

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11. Seth, Janice, and Lori each borrow 5000 for five years at a nominal interest rate of 12%, compounded semiannually. Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years. Janice pays interest at the end of every six month period as it accrues and the principal at the end of five years. Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

- a. 8718 b. 8728 c. 8738 d. 8748 e. 8758

12. A 10-year loan of 2000 is to be repaid with payments at the end of each year.

It can be repaid under the following two options:

- (i) Equal annual payments at an annual effective rate of 8.07%.
(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective rate of i .

The sum of the payments under option (i) equals the sum of the payment under option (ii)

Determine i .

- a. 8.75% b. 9.00% c. 9.25% d. 9.50% e. 9.75%

13. At an annual effective interest rate of i , $i > 0$, both of the following annuities have a present value of X .

- (i) A 20-year annuity-immediate with annual payments of 55
(ii) A 30-year annuity immediate with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years.

Calculate X .

- a. 575 b. 585 c. 595 d. 605 e. 615

Section 2 Key

1. B

2. A

3. E

4. B

5. C

6. D

7. D

8. D

9. C

10. D

11. D

12. B

13. A