Exam FM/2 Mock Exam (March 23, 2010) Name:______________________________

1. Al buys a 1-year 50-strike call option on XYZ stock for $7.00 and sells a 1-year 55-strike call option on the same stock for $3.50. The risk free interest rate is 3% compounded continuously. Determine the value of XYZ's stock in 1 year in order for Al to breakeven.

(A) 51.03 (B) 51.39 (C) 52.50 (D) 53.61 (E) 53.97

2. A loan of 12 is to be repaid with payments of 10 at the end of 3 years and 5 at the end of 6 years. Calculate the simple discount rate that is being charged on the loan at the time the loan was made.

(A) 5.00% (B) 5.25% (C) 5.50% (D) 5.75% (E) 6.00%

3. The ABC Company has two liabilities: 11,400 in one year and 20,800 in two years. They have two types of bonds to use to exactly match these liabilities.

Bond 1 is a 1000 face value 2-year bond with 4% annual coupons redeemable at par and can be bought to yield 5%.

Bond 2 is a 1000 face value 1-year bond with 6% annual coupons redeemable at par and can be bought to yield 4%.

Determine how much it will cost the ABC Company to exactly match it's liabilities.

(A) 29,820 (B) 29,830 (C) 30,070 (D) 30,080 (E) 30,090

4. On July 1, 2008, a person invested 500 in a fund for which the force of interest at time *t* is given by $\delta_t = \frac{3+t}{30}$, where *t* is the number of years since January 1, 2007. Determine the accumulated value of the investment on January 1, 2009.

(A) 530 (B) 535 (C) 540 (D) 545 (E) 550

5. Fund X starts with 1000 and accumulates with force of interest $\delta_t = \frac{1}{15-t}$, for $0 < t < 15$.

Fund Y starts with 1000 and accumulates with an interest rate of 8% per annum compounded semiannually for the first three years and an effective interest rate of *i* per annum thereafter.

The amount in Fund X equals the amount in Fund Y at the end of four years. Calculate *i*.

(A) 7.50% (B) 7.75% (C) 8.00% (D) 8.25% (E) 8.50%

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6. You are given:

(i) the annual yield rate on a zero-coupon bond with duration of 1 year is 3%.
(ii) the annual yield rate on a zero-coupon bond with duration of 2 years is 5%.
(iii) the annual yield rate on a zero-coupon bond with duration of 3 years is 6%.

You would like to swap payments of 100, 200, and 300, due at the end of years 1, 2, and 3, respectively, for a level set of payments. Determine the swap price.

(A) 185 (B) 190 (C) 195 (D) 200 (E) 205

7. An account is credited interest using 10% simple interest rate from the date of each deposit into the account. Monthly payments of 100 are deposited into this account. Calculate the accumulated value of the account immediately after the 24th deposit.

(A) 2630 (B) 2645 (C) 2650 (D) 2660 (E) 2665

8. An annuity pays 3 at the beginning of each 3 year period for 30 years. Find the accumulated value of the annuity just after the final payment, using $i^{(2)} = .06$.

(A) 35 (B) 45 (C) 60 (D) 75 (E) 90

9. Let P_0 be the premium for an at-the-money put option and let C_0 be the premium for an at-the-money call option. Which of the following statements is true?

I. $P_0 < C_0$ if the risk-free interest rate is positive
II. $P_0 > C_0$ if the risk-free interest rate is positive
III. $P_0 = C_0$ if the risk-free interest rate is zero

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III

10. A homeowner purchases homeowner's insurance. From the perspective of the homeowner, the purchase of this insurance is equivalent to which of the following positions?

(A) long put (B) long call (C) short put (D) short call (E) none of these

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11. Joe buys a put with strike price 40 and buys a call with strike price 50 on company XYZ stock which is currently trading at 45. Which situation(s) will Joe profit from his position?

I. Company XYZ stock increases significantly
II. Company XYZ stock decreases significantly
III. Company XYZ stock increases slightly
IV. Company XYZ stock decreases slightly

(A) I and II
(B) I and III
(C) II and IV
(D) III and IV
(E) The correct answer is not given.

12. Esther invests 100 at the end of each year for 12 years at an annual effective interest rate of i. The interest payments are reinvested at an annual effective rate of 5%. The accumulated value at the end of 12 years is 1748.40. Calculate i.

(A) 6% (B) 7% (C) 8% (D) 9% (E) 10%

13. Chris makes annual deposits into a bank account at the beginning of each year for 20 years. Chris' initial deposit is equal to 100, with each subsequent deposit k% greater than the previous year's deposit. The bank credits interest at an annual effective interest rate of 5%.

At the end of 20 years, the accumulated value in Chris' account is equal to 7276.35.

Given $k > 5$, calculate k.

(A) 8.06 (B) 8.21 (C) 8.36 (D) 8.51 (E) 8.6

14. A borrower is repaying a loan of 300,000 by the sinking fund method. The sinking fund earns an annual effective interest rate of 6.75%. Payments of $22,520 are made at the end of each year for 20 years to repay the loan. These payments consist of both the interest payment to the lender and also the sinking fund deposit.

What is the annual effective interest rate paid to the lender of the loan?

(A) 4.0% (B) 4.5% (C) 5.0% (D) 5.5% (E) 6.0%

15. A loan of 1000 is being repaid in ten years by semiannual installments of 50, plus interest on the unpaid balance at 4% per annum compounded semiannually. The installments and interest payments are reinvested at 5% per annum compounded semiannually. Calculate the annual effective yield rate of the loan.

(A) 4.6% (B) 4.8% (C) 5.0% (D) 5.2% (E) 5.4%

16. A 1000 face value 20-year 6% bond with semiannual coupons is bought to yield 5%, compounded semiannually. The redemption value is 700. The coupons are reinvested at a nominal annual rate of 6%, compounded semiannually. Determine the purchaser's annual effective yield rate over the 20-year period.

(A) 5.1% (B) 5.3% (C) 5.5% (D) 5.7% (E) 5.9%

17. A 30-year 10,000 bond that pays 3% annual coupons matures at par. It is purchased to yield 5% for the first 15 years and 4% thereafter. Calculate the amount for accumulation of discount for year 8.

(A) 78 (B) 83 (C) 88 (D) 93 (E) 98

18. A portfolio consists of two bonds:

Bond A is a n-year bond with a duration of 17 years and was bought for 885.

Bond B is a 10 year zero-coupon bond that cost 1115.

Determine the modified duration of the portfolio. The interest rate used in all calculation is 6% compounded annually.

(A) 12.4 (B) 12.7 (C) 13.1 (D) 13.5 (E) 13.9

19. You are given 1-year, 2-year, and 3-year spot rates of 4%, 5%, and 6%, respectively.

Calculate the annual yield rate for 3-year 5% annual coupon bonds implied by these spot rates.

(A) 5.1% (B) 5.3% (C) 5.5% (D) 5.7% (E) 5.9%

20. Calculate the duration of a common stock that pays dividends at the end of each year into perpetuity. Assume that the dividends increase by 2% each year and that the effective rate of interest is 5%.

(A) 27 (B) 35 (C) 44 (D) 52 (E) 58

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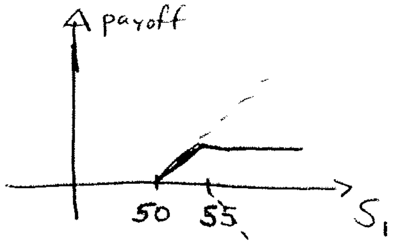
Key:

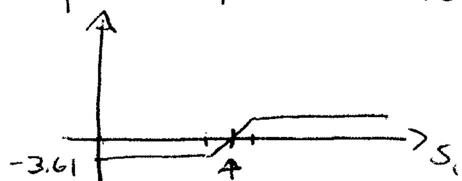
1) long call (50) + short call (55)



$IC = 7 - 3.50 = 3.50$

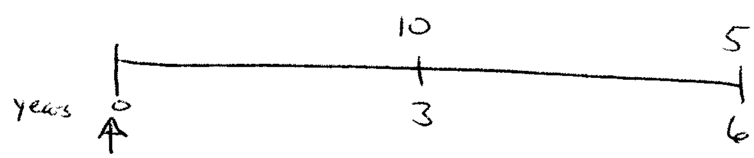
$AV_{IC} = 3.5\,e^{.03} \doteq 3.61$

profit = payoff − AV_{IC}



break-even = 53.61

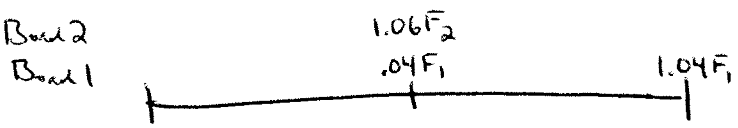
2)



d = simple discount rate

$$12 = 10(1-3d) + 5(1-6d)$$
$$12 = 15 - 60d \Rightarrow d = \frac{3}{60} = .05$$

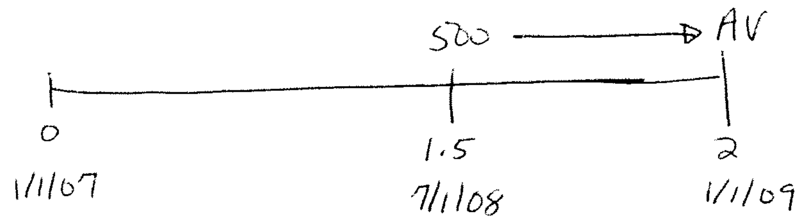
3) F_1 = face amount of bond 1 ; F_2 = face amount of bond 2



$$\left.\begin{array}{r} 1.06F_2 + .04F_1 = 11400 \\ 1.04F_1 = 20800 \end{array}\right\} \Rightarrow F_1 = 20000 \;\&\; F_2 = 10000$$

$$P_1 = \text{Price of Bond } 1 = 800\,a_{\overline{2}|.05} + 20000\,v^2_{.05} = 19628$$
$$P_2 = 1.06(10000)\,v_{.04} = 10192$$
$$\therefore \Sigma = 19628 + 10192 = 29820$$

4)



$$AV = 500\, e^{\int_{1.5}^{2} \frac{3+t}{30}\,dt} = 500\, e^{\frac{1}{30}\left(3t+\frac{t^2}{2}\right)\Big|_{1.5}^{2}} = 500 e^{.0791\overline{6}}$$

$$AV \doteq 540$$

5) $X: \quad a(t) = \left(\frac{15-t}{15}\right)^{-1} = \frac{15}{15-t}$

$$\left.\begin{array}{l} AV_4^X = 1000 \cdot a(4) = 1000 \cdot \frac{15}{11} \\ AV_4^Y = 1000(1.04)^6(1+i) \end{array}\right\} = \quad \therefore\ 1+i = \frac{15}{11(1.04)^6}$$

$$\Longrightarrow i \doteq .0775$$

6) 1-year spot rate $S_1 = .03$

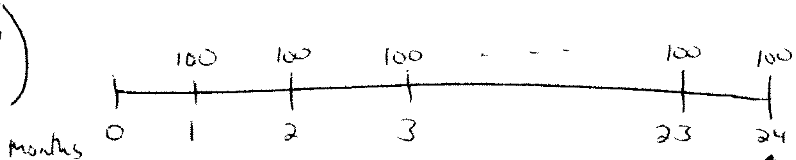
$S_2 = .05$

$S_3 = .06$

$$\frac{100}{1.03} + \frac{200}{(1.05)^2} + \frac{300}{(1.06)^3} = R\left(\frac{1}{1.03} + \frac{1}{(1.05)^2} + \frac{1}{(1.06)^3}\right)$$

$$\Longrightarrow R \doteq 195$$

7)

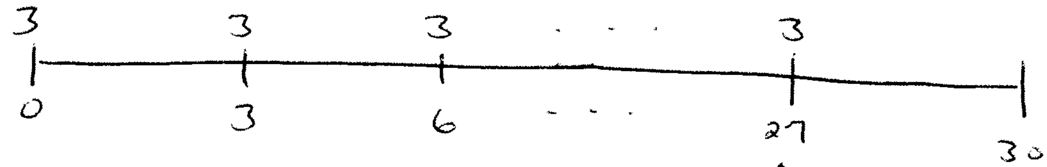


$$AV = 100 + 100\left(1+\frac{i}{12}\right) + 100\left(1+2\frac{i}{12}\right) + \cdots + 100\left(1+23\frac{i}{12}\right)$$

$$AV = 100(24) + 100\frac{i}{12}(1+2+3+\cdots+23) = 2400 + 100\left(\frac{.10}{12}\right)\left(\frac{23\cdot 24}{2}\right)$$

$$AV = 2630$$

8)



$AV = 3\, s_{\overline{10}|j}$

j = 3-year eir

$1+j = (1.03)^6 \Rightarrow j \doteq .194$

$AV = 3\, s_{\overline{10}|.194} \doteq 75$

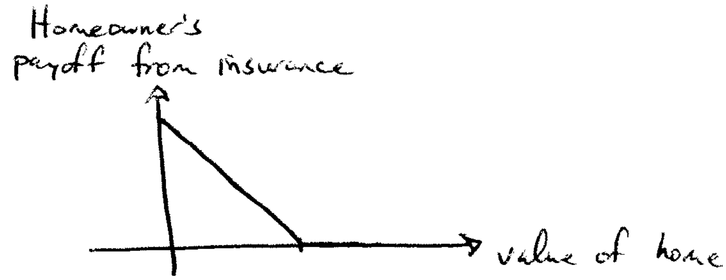
9) Put - Call Parity : $(K = S_0)$ $\quad C_0 - P_0 = S_0(1 - v^T)$

If $r > 0$, then $v^T < 1$ and so $C_0 > P_0$

If $r = 0$, then $v^T = 1$ and so $C_0 = P_0$

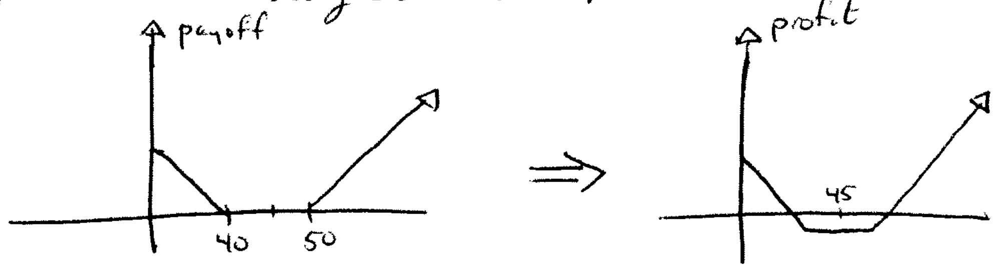
I & III are true

10)



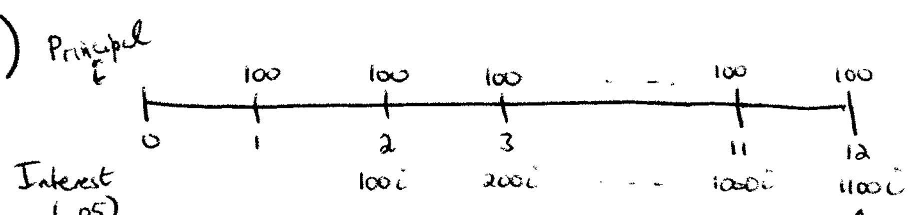
long put

11) long put (40) + long call (50)



Joe profits from a significant increase or decrease in stock price,
I & II are true

12)

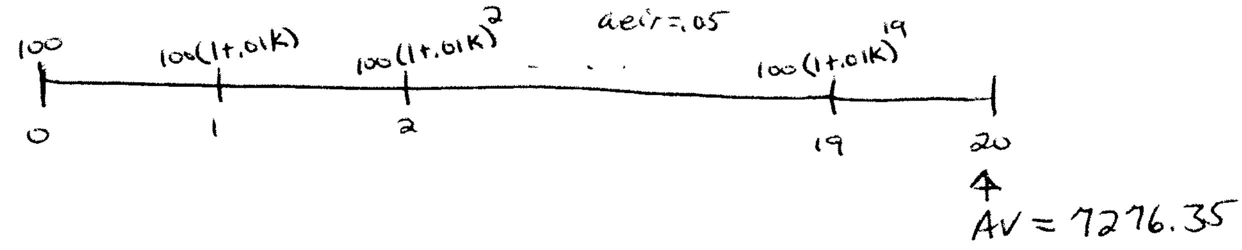


$AV = 100(12) + 100i \cdot (Is)_{\overline{11}|.05}$

$AV = 1200 + 100i \cdot \dfrac{\ddot{s}_{\overline{11}|.05} - 11}{.05} = 1748.40$

$\Rightarrow i \doteq .07$

13)



$$7276.35 \stackrel{\text{VEP}}{=} 100(1.05)^{20} + 100(1+.01k)(1.05)^{19} + \cdots + 100(1+.01k)^{19}(1.05)$$

$$= 100(1.05)^{20}\left[1 + \frac{1+.01k}{1.05} + \cdots + \left(\frac{1+.01k}{1.05}\right)^{19}\right]$$

geometric $r = \frac{1+.01k}{1.05} > 1$ since $k > 5$

$$= 100(1.05)^{20} \cdot s_{\overline{20}|j} \quad j = r-1$$

$$7276.35 = 100(1.05)^{20} \cdot s_{\overline{20}|j} \implies j \doteq .032 \quad \text{(Use calculator TVM)}$$

$$\therefore \quad r = \frac{1+.01k}{1.05} = 1.032 \implies k \doteq 8.36$$

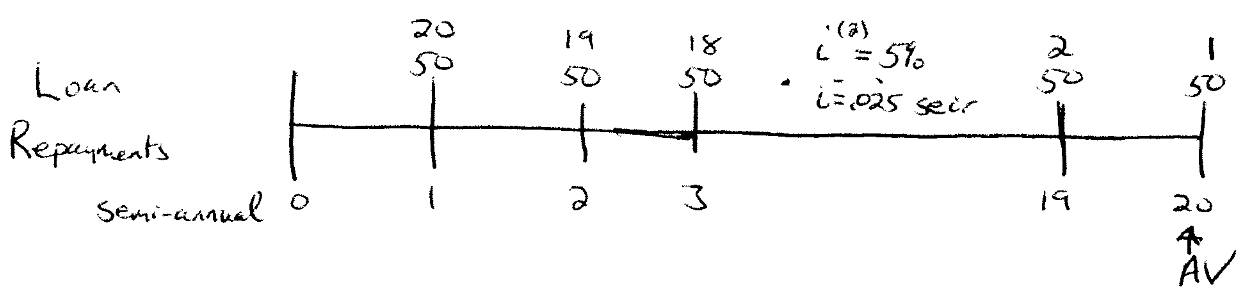
14)

$$R_{SF} = \frac{300000}{s_{\overline{20}|.0675}} \implies R_{SF} = 7520$$

$$R_{Total} = 22520 \implies R_I = 15000 = 300000 \cdot i$$

$$\implies i = .05$$

15)

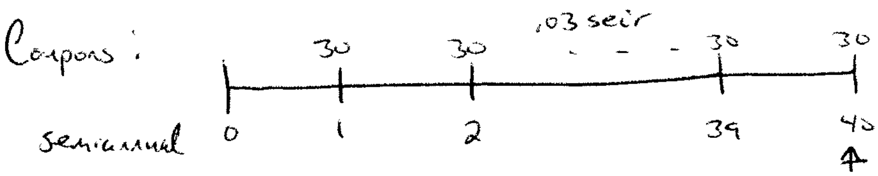


$$AV = 50\, s_{\overline{20}|.025} + (Ds)_{\overline{20}|.025}$$

$$= 50\, s_{\overline{20}|.025} + \frac{20(1.025)^{20} - s_{\overline{20}|.025}}{.025} \doteq 1566.34$$

$$\therefore \quad 1000(1+j)^{10} = 1566.34 \implies j \doteq 4.6\%$$

16) $\text{Price} = P = 30\, a_{\overline{40}|.025} + 700\, v^{40}_{.025} \doteq 1013.78$



$AV = 30\, s_{\overline{40}|.03} = 2262.04$

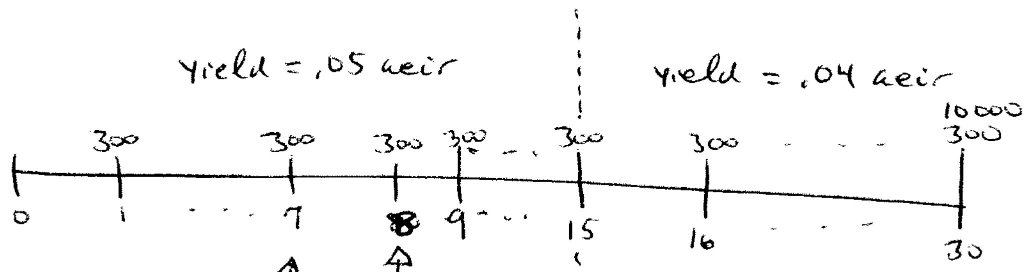
$+ \ 700$ (Redemption)

2962.04

$\therefore\ 1013.78(1+i)^{20} = 2962.04$

$\Rightarrow i \doteq 5.59\%$

17)



$BV_{15} = 300\, a_{\overline{15}|.04} + 10000\, v^{15}_{.04}$

$= 8888.16$

$BV_8 = 300\, a_{\overline{7}|.05} + 8888.16\, v^{7}_{.05} \doteq 8052.56$

$BV_7 = 300\, a_{\overline{8}|.05} + 8888.16\, v^{8}_{.05} \doteq 7954.82$

Accumulation of discount for year 8 is

$BV_8 - BV_7 \doteq 98$

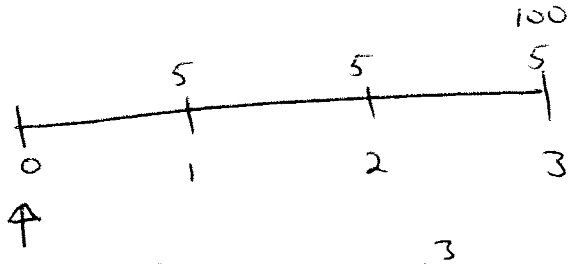
18) $\text{Mod D (Bond A)} = 17\, v_{.06} = 16.0377$

$\text{Mod D (Bond B)} = 10\, v_{.06} = 9.4340$

$\therefore \text{Mod D (Portfolio)} = \frac{885}{885+1115}(16.0377) + \frac{1115}{885+1115}(9.4340)$

$\doteq 12.4$

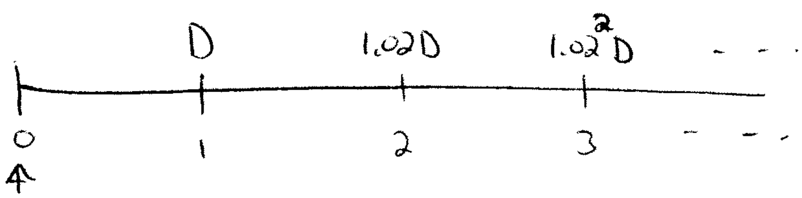
19) Per 100 of face value, we have



$$P = 5\,a_{\overline{3}|i} + 100\,v_i^3 = \frac{5}{1.04} + \frac{5}{(1.05)^2} + \frac{105}{(1.06)^3}$$

$\Longrightarrow i \doteq 5.9\%$ (Use calculator TVM)

20)



$$\text{Mac}\,D = \frac{Dv + 2\cdot 1.02D\cdot v^2 + 3\cdot 1.02^2 D\cdot v^3 + \cdots}{Dv + 1.02Dv^2 + 1.02^2 Dv^3 + \cdots}$$

$$= \frac{\cancel{Dv}\left(1 + 2(1.02v) + 3(1.02v^2) + \cdots\right)}{\cancel{Dv}\left(1 + 1.02v + (1.02v)^2 + \cdots\right)}$$

Let $X = 1.02v = \frac{1.02}{1.05}$ (like a new v)

Then $$\text{Mac}\,D = \frac{1 + 2X + 3X^2 + \cdots}{1 + X + X^2 + \cdots} = \frac{(I\ddot{a})_{\overline{\infty}|}}{\ddot{a}_{\overline{\infty}|}}$$

Note: $$(I\ddot{a})_{\overline{\infty}|} = \frac{\ddot{a}_{\overline{\infty}|} - \cancel{\infty v^{\infty}}^{\,\to 0}}{d} = \frac{\ddot{a}_{\overline{\infty}|}}{d}$$

$$\therefore\ \text{Mac}\,D = \frac{\left(\frac{\ddot{a}_{\overline{\infty}|}}{d}\right)}{\ddot{a}_{\overline{\infty}|}} = \frac{1}{d} = \frac{1}{1 - v_{new}} = \frac{1}{1 - X} = 35$$