

All questions carry equal weight. State answers clearly and carefully, and justify all assertions with proofs or counterexamples. You may not use any books or notes.

(1) Describe 8 non-isomorphic groups of order 36.

(2) Let  $G$  be a subgroup of  $S_6$ , and assume that  $G$  has an element of order 6. Show that  $G$  contains a normal subgroup of index 2.

(3) How many distinct isomorphisms are there from  $S_4$  to itself?

(4) Let  $G$  be a finite group, and let  $S$  be the set of elements  $g \in G$  such that  $g = g^{-1}$ . Show that  $|G|$  is even if and only if  $S$  has more than one element.