1) Prove that if $f \in \mathbb{Z}[x]$ is primitive and $g \in \mathbb{Z}[x]$ divides f in $\mathbb{Z}[x]$, then either g or $-g$ is also primitive.

2) Find wether or not the following polynomials are irreducible over $\mathbb{Q}[x]$.

(a) $f_1(x) = x^4 + x^3 + x - 6$

(b) $f_2(x) = x^6 - 2x^5 + 14x^2 - 8x + 34$

(c) $f_3(x) = 100x^3 - x + 2008$

(d) $f_4(x) = x^4 + x^3 + x^2 + x + 1$

3) Let F be a field. We say that $\alpha \in F$ is a *multiple root* of $f(x) \in F[x]$ if $f(x) = (x - \alpha)^2 \cdot g(x)$, for some $g \in F[x]$.

(a) Prove that if α is a multiple root of f, then $f(\alpha) = f'(\alpha) = 0$, where $f'(x)$ is the derivative of $f(x)$ [as in calculus]. [Note that all calculus formulas for derivatives hold for polynomials.]

(b) Prove that if $f(x) \in F[x]$ is irreducible, then $f(x)$ has no multiple roots in any extension of F, as long as $f'(x) \neq 0$. [**Hint:** What's the greatest common divisor of $f(x)$ and $f'(x)$?]

4) Let R be a UFD and let P be a non-zero *prime* ideal of R such that if P' is another prime ideal, with $(0) \subsetneq P' \subseteq P$, then $P' = P$. Prove that P is principal. [**Hint:** $P = (p)$ is prime iff p is what?]

5) Maximal ideals of polynomial rings with complex coefficients.

(a) Prove that if I is an ideal of $\mathbb{C}[x, y]$ and M is a maximal ideal containing I, then there is a point (a, b) such that for all $f(x, y) \in I$, we have $f(a, b) = 0$.

[**Observation:** This statement is also true for n variables (with an analogous solution).]

(b) Let $I = (3x - y - 2, y - x^2)$ be an ideal of $\mathbb{C}[x, y]$. Find *all* maximal ideals of $\mathbb{C}[x, y]$ that contain I.