1) Let R be a ring and I be an ideal of R.

(a) Prove that if J is an ideal of R containing I, then $\bar{J} \stackrel{\text{def}}{=} \{\bar{a} \in R/I \ : \ a \in J\}$ is an ideal of R/I.

(b) Prove that if $\bar{J}'$ is an ideal of R/I, then $J' \stackrel{\text{def}}{=} \{a \in R \ : \ \bar{a} \in \bar{J}'\}$ is an ideal of R containing I.

2) Let R be a commutative ring with identity and $a \in R$ such that $a^{n-1} \neq 0$, but $a^n = 0$, for some positive integer n. Prove that $R[x]/(ax-1) = \{\bar{0}\}$, i.e., it is the *zero ring.*

3) Let R be an integral domain, F be its field of fractions [or quotient field], and K be field such that $R \subseteq K$. Prove that there is an *injective homomorphism* $\phi : F \to K$, such that for all $a \in R$, $\phi\left(\frac{a}{1}\right) = a$. [**Hint:** To start, you need to find the formula for ϕ. Think of the most natural way of seeing an element of F inside of K, remembering that the image is contained in a *field*. Also, you will have to show that your formula is well defined, i.e., if $\frac{a}{b} = \frac{c}{d}$, then $\phi\left(\frac{a}{b}\right) = \phi\left(\frac{c}{d}\right)$.]

4) Prove that $\mathbb{Z}[\mathrm{i}\sqrt{3}]/(2-\mathrm{i}\sqrt{3}) \cong \mathbb{Z}/7\mathbb{Z}$.